

CHAPTER 2

MATERIAL AND ENERGY BALANCES

Material quantities, as they pass through food processing operations, can be described by material balances. Such balances are statements on the conservation of mass. Similarly, energy quantities can be described by energy balances, which are statements on the conservation of energy. If there is no accumulation, what goes into a process must come out. This is true for batch operation. It is equally true for continuous operation over any chosen time interval.

Material and energy balances are very important in the food industry. Material balances are fundamental to the control of processing, particularly in the control of yields of the products. The first material balances are determined in the exploratory stages of a new process, improved during pilot plant experiments when the process is being planned and tested, checked out when the plant is commissioned and then refined and maintained as a control instrument as production continues. When any changes occur in the process, the material balances need to be determined again.

The increasing cost of energy has caused the food industry to examine means of reducing energy consumption in processing. Energy balances are used in the examination of the various stages of a process, over the whole process and even extending over the total food production system from the farm to the consumer's plate.

Material and energy balances can be simple, at times they can be very complicated, but the basic approach is general. Experience in working with the simpler systems such as individual unit operations will develop the facility to extend the methods to the more complicated situations, which do arise. The increasing availability of computers has meant that very complex mass and energy balances can be set up and manipulated quite readily and therefore used in everyday process management to maximise product yields and minimise costs.

BASIC PRINCIPLES

If the unit operation, whatever its nature is seen as a whole it may be represented diagrammatically as a box, as shown in Fig. 2.1. The mass and energy going into the box must balance with the mass and energy coming out.

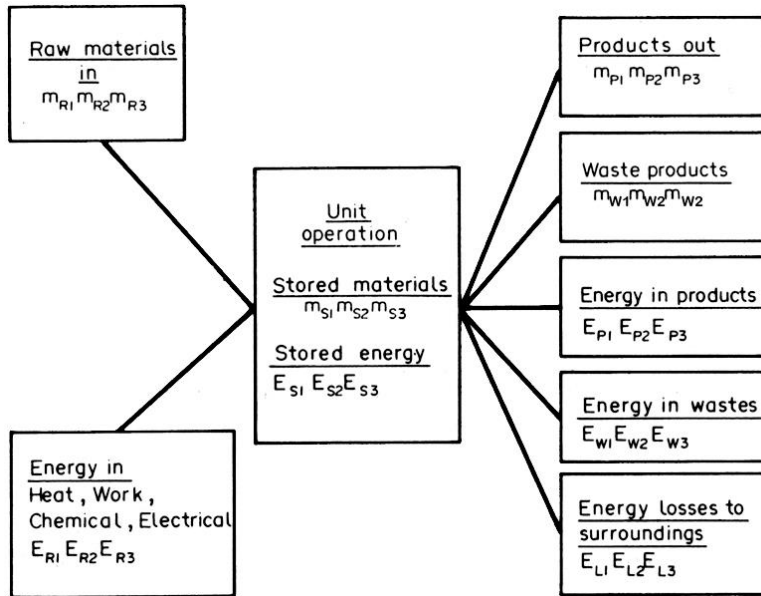


Figure 2.1. Mass and energy balancejoeperreau@aol.com

The law of conservation of mass leads to what is called a mass or a material balance.

$$\text{Mass In} = \text{Mass Out} + \text{Mass Stored}$$

$$\text{Raw Materials} = \text{Products} + \text{Wastes} + \text{Stored Materials.}$$

$$\sum m_R = \sum m_P + \sum m_W + \sum m_S$$

(where Σ (sigma) denotes the sum of all terms).

$$\sum m_R = m_{R1} + m_{R2} + m_{R3} = \text{Total Raw Materials.}$$

$$\sum m_P = m_{P1} + m_{P2} + m_{P3} = \text{Total Products.}$$

$$\sum m_W = m_{W1} + m_{W2} + m_{W3} = \text{Total Waste Products.}$$

$$\sum m_S = m_{S1} + m_{S2} + m_{S3} = \text{Total Stored Products.}$$

If there are no chemical changes occurring in the plant, the law of conservation of mass will apply also to each component, so that for component A:

$$m_A \text{ in entering materials} = m_A \text{ in the exit materials} + m_A \text{ stored in plant.}$$

For example, in a plant that is producing sugar, if the total quantity of sugar going into the plant is not equalled by the total of the purified sugar and the sugar in the waste liquors, then there is something wrong. Sugar is either being burned (chemically changed) or accumulating in the plant or else it is going unnoticed down the drain somewhere. In this case:

$$(m_{AR}) = (m_{AP} + m_{AW} + m_{AS} + m_{AU})$$

where m_{AU} is the unknown loss and needs to be identified. So the material balance is now:

$$\text{Raw Materials} = \text{Products} + \text{Waste Products} + \text{Stored Products} + \text{Losses}$$

where Losses are the unidentified materials.

Just as mass is conserved, so is energy conserved in food-processing operations. The energy coming into a unit operation can be balanced with the energy coming out and the energy stored.

$$\begin{aligned} \text{Energy In} &= \text{Energy Out} + \text{Energy Stored} \\ \Sigma E_R &= \Sigma E_P + \Sigma E_W + \Sigma E_L + \Sigma E_S \end{aligned}$$

where:

$$\begin{aligned} \Sigma E_R &= E_{R1} + E_{R2} + E_{R3} + \dots &= \text{Total Energy Entering} \\ \Sigma E_P &= E_{P1} + E_{P2} + E_{P3} + \dots &= \text{Total Energy Leaving with Products} \\ \Sigma E_W &= E_{W1} + E_{W2} + E_{W3} + \dots &= \text{Total Energy Leaving with Waste Materials} \\ \Sigma E_L &= E_{L1} + E_{L2} + E_{L3} + \dots &= \text{Total Energy Lost to Surroundings} \\ \Sigma E_S &= E_{S1} + E_{S2} + E_{S3} + \dots &= \text{Total Energy Stored} \end{aligned}$$

Energy balances are often complicated because forms of energy can be interconverted, for example mechanical energy to heat energy, but overall the quantities must balance.

MATERIAL BALANCES

The first step is to look at the three basic categories: materials in, materials out and materials stored. Then the materials in each category have to be considered whether they are to be treated as a whole, a gross mass balance, or whether various constituents should be treated separately and if so what constituents. To take a simple example, it might be to take dry solids as opposed to total material; this really means separating the two groups of constituents, non-water and water. More complete dissection can separate out chemical types such as minerals, or chemical elements such as carbon. The choice and the detail depend on the reasons for making the balance and on the information that is required. A major factor in industry is, of course, the value of the materials and so expensive raw materials are more likely to be considered than cheaper ones, and products than waste materials.

Basis and Units

Having decided which constituents need consideration, the basis for the calculations has to be decided. This might be some mass of raw material entering the process in a batch system, or some mass per hour in a continuous process. It could be: some mass of a particular predominant constituent, for example mass balances in a bakery might be all related to 100 kg of flour entering; or some unchanging constituent, such as in combustion calculations with air where it is helpful to relate everything to the inert nitrogen component; or carbon added in the nutrients in a

fermentation system because the essential energy relationships of the growing micro-organisms are related to the combined carbon in the feed; or the essentially inert non-oil constituents of the oilseeds in an oil-extraction process. Sometimes it is unimportant what basis is chosen and in such cases a convenient quantity such as the total raw materials into one batch or passed in per hour to a continuous process are often selected. Having selected the basis, then the units may be chosen such as mass, or concentrations which can be by weight or can be molar if reactions are important.

Total mass and composition

Material balances can be based on total mass, mass of dry solids, or mass of particular components, for example protein.

EXAMPLE 2.1. Constituent balance of milk

Skim milk is prepared by the removal of some of the fat from whole milk. This skim milk is found to contain 90.5% water, 3.5% protein, 5.1% carbohydrate, 0.1% fat and 0.8% ash. If the original milk contained 4.5% fat, calculate its composition, assuming that fat only was removed to make the skim milk and that there are no losses in processing.

Basis: 100 kg of skim milk. This contains, therefore, 0.1 kg of fat. Let the fat which was removed from it to make skim milk be x kg.

$$\begin{aligned} \text{Total original fat} &= (x + 0.1) \text{ kg} \\ \text{Total original mass} &= (100 + x) \text{ kg} \end{aligned}$$

and as it is known that the original fat content was 4.5% so

$$\frac{x + 0.1}{100 + x} = 0.045$$

$$\begin{aligned} \text{whence } x + 0.1 &= 0.045(100 + x) \\ x &= 4.6 \text{ kg} \end{aligned}$$

So the composition of the whole milk is then:

$$\begin{aligned} \text{fat} = 4.5\% , \text{ water } \frac{90.5}{104.6} = 86.5\% , \text{ protein } = \frac{3.5}{104.6} = 3.3\% , \text{ carbohydrate } = \frac{5.1}{104.6} = 4.9\% \\ \text{and ash} = 0.8\% \end{aligned}$$

Total composition: water 86.5%, carbohydrate 4.9%, fat 4.5%, protein 3.3%, ash 0.8%

Concentrations

Concentrations can be expressed in many ways: weight/ weight (w/w), weight/volume (w/v), molar concentration (M), mole fraction. The weight/weight concentration is the weight of the solute divided by the total weight of the solution and this is the fractional form of the percentage composition by weight. The weight volume concentration is the weight of solute in the total volume of the solution. The molar concentration is the number of moles (molecular weights) of

the solute in a volume of the solution, in this book expressed as kg mole in 1 m^3 of the solution. The mole fraction is the ratio of the number of moles of the solute to the total number of moles of all species present in the solution. Notice that in process engineering, it is usual to consider kg moles and in this book the term mole means a mass of the material equal to its molecular weight in kilograms. In this book, percentage signifies percentage by weight (w/w) unless otherwise specified.

EXAMPLE 2.2. Concentrations

A solution of common salt in water is prepared by adding 20 kg of salt to 100 kg of water, to make a liquid of density 1323 kg m^{-3} . Calculate the concentration of salt in this solution as a (a) weight fraction, (b) weight/volume fraction, (c) mole fraction, (d) molal concentration.

(a) Weight fraction:

$$\frac{20}{100 + 20} = 0.167$$

$$\underline{\% \text{ weight/weight}} = 16.7\%$$

(b) Weight/volume:

A density of 1323 kg m^{-3} means that 1 m^3 of solution weighs 1323kg, but 1323kg of salt solution contains:

$$\frac{20}{100 + 20} \times 1323 \text{ kg salt} = 220.5 \text{ kg salt m}^{-3}$$

and so 1 m^3 solution contains 220.5 kg salt.

$$\text{Weight/volume fraction} = \frac{220.5}{1000} = 0.2205.$$

$$\text{and so } \underline{\% \text{ weight/volume}} = 22.1\%$$

$$\text{(c) Moles of water} = \frac{100}{18} = 5.56$$

$$\text{Moles of salt} = \frac{20}{58.5} = 0.34.$$

$$\text{Mole fraction of salt} = \frac{0.34}{5.56 + 0.34}$$

and so mole fraction of salt = 0.058.

(d) The molar concentration (M) is $220.5/58.5 = 3.77 \text{ moles in } 1 \text{ m}^3$.

Note that the mole fraction can be approximated by the (moles of salt/moles of water) as the number of moles of water are dominant, that is the mole fraction is close to $0.34/5.56 = 0.061$. As the solution becomes more dilute, this approximation improves and generally for dilute

solutions the mole fraction of solute is a close approximation to the moles of solute/moles of solvent.

In solid/liquid mixtures, all these methods can be used but in solid mixtures the concentrations are normally expressed as simple weight fractions.

With gases, concentrations are primarily measured in weight concentrations per unit volume, or as partial pressures. These can be related through the gas laws. Using the gas law in the form:

$$pV = nRT$$

where p is the pressure, V the volume, n the number of moles, T the absolute temperature, and R the gas constant which is equal to $0.08206 \text{ m}^3 \text{ atm mole}^{-1} \text{ K}^{-1}$, the molar concentration of a gas is then

$$n/V = p/RT$$

and the weight concentration is then nM/V where M is the molecular weight of the gas.

The SI unit of pressure is N m^{-2} called the Pascal (Pa). As this is of inconvenient size for many purposes, standard atmospheres (atm) are often used as pressure units, the conversion being $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$, or very nearly $1 \text{ atm} = 100 \text{ kPa}$.

EXAMPLE 2.3. Air composition

If air consists of 77% by weight of nitrogen and 23% by weight of oxygen calculate:

- the mean molecular weight of air,
- the mole fraction of oxygen,
- the concentration of oxygen in mole m^{-3} and kg m^{-3} if the total pressure is 1.5 atmospheres and the temperature is 25°C .

- (a) Taking the basis of 100 kg of air: it contains $\frac{77}{28}$ moles of N_2 and $\frac{23}{32}$ moles of O_2

$$\text{Total number of moles} = 2.75 + 0.72 = 3.47 \text{ moles}$$

$$\text{So mean molecular weight of air} = \frac{100}{3.47} = 28.8.$$

$$\underline{\text{Mean molecular weight of air} = 28.8}$$

- (b) The mole fraction of oxygen = $\frac{0.72}{2.75 + 0.72} = \frac{0.72}{3.47} = 0.21$

$$\underline{\text{Mole fraction of oxygen} = 0.21}$$

(Note this is also the volume fraction)

(c) In the gas equation, where n is the number of moles present: the value of R is $0.08206 \text{ m}^3 \text{ atm mole}^{-1} \text{ K}^{-1}$ and at a temperature of $25^\circ\text{C} = 25 + 273 = 298 \text{ K}$, and where $V = 1 \text{ m}^3$

$$pV = nRT$$

and so

$$1.5 \times 1 = n \times 0.08206 \times 298$$

$$n = 0.061 \text{ moles}$$

weight of air in $1 \text{ m}^3 = n \times \text{mean molecular weight}$

$$= 0.061 \times 28.8$$

$$= 1.76 \text{ kg}$$

and of this 23% is oxygen, so weight of oxygen in 1 m^3

$$= 0.23 \times 1.76$$

$$= 0.4 \text{ kg}$$

$$\frac{\text{Concentration of oxygen}}{\text{or } \frac{0.4}{32}} = \frac{0.4 \text{ kg m}^{-3}}{0.013 \text{ mole m}^{-3}}$$

When a gas is dissolved in a liquid, the mole fraction of the gas in the liquid can be determined by first calculating the number of moles of gas using the gas laws, treating the volume as the volume of the liquid, and then calculating the number of moles of liquid directly.

EXAMPLE 2.4. Carbonation of a soft drink

In the carbonation of a soft drink, the total quantity of carbon dioxide required is the equivalent of 3 volumes of gas to one volume of water at 0°C and atmospheric pressure. Calculate (a) the mass fraction and (b) the mole fraction of the CO_2 in the drink, ignoring all components other than CO_2 and water.

Basis 1 m^3 of water = 1000 kg.
Volume of carbon dioxide added = 3 m^3

From the gas equation $pV = nRT$

And so

$$1 \times 3 = n \times 0.08206 \times 273.$$

$$n = 0.134 \text{ moles}$$

Molecular weight of carbon dioxide = 44
And so weight of carbon dioxide added = $0.134 \times 44 = 5.9 \text{ kg}$

(a) Mass fraction of carbon dioxide in drink = $5.9 / (1000 + 5.9) = 5.9 \times 10^{-3}$

(b) Mole fraction of carbon dioxide in drink = $0.134 / (1000/18 + 0.134) = 2.41 \times 10^{-3}$

Types of Process Situations

Continuous processes

In continuous processes, time also enters into consideration and the balances are related to unit time. Consider a continuous centrifuge separating whole milk into skim milk and cream. If the material hold-up in the centrifuge is constant both in mass and in composition, then the quantities of the components entering and leaving in the different streams in unit time are constant and a materials balance can be written on this basis. Such an analysis assumes that the process is in a steady state, that is flows and quantities held up in vessels do not change with time.

EXAMPLE 2.5. Materials balance in continuous centrifuging of milk

If 35,000kg of whole milk containing 4% fat is to be separated in a 6 hour period into skim milk with 0.45% fat and cream with 45% fat, what are the flow rates of the two output streams from a continuous centrifuge which accomplishes this separation?

Basis 1 hour's flow of whole milk

Mass in

$$\text{Total mass} = \frac{35,000}{6} = 5833 \text{ kg}$$

$$\text{Fat} = 5833 \times 0.04 = 233 \text{ kg}$$

$$\text{And so water plus solids-not-fat} = 5600 \text{ kg}$$

Mass out

Let the mass of cream be x kg then its total fat content is $0.45x$. The mass of skim milk is $(5833 - x)$ and its total fat content is $0.0045(5833 - x)$.

Materials balance on fat:

$$\text{Fat in} = \text{Fat out}$$

$$5833 \times 0.04 = 0.0045(5833 - x) + 0.45x$$

$$\text{and so } x = 465 \text{ kg}$$

$$\underline{\text{So that the flow of cream is } 465 \text{ kg h}^{-1} \text{ and skim milk } (5833 - 465) = 5368 \text{ kg h}^{-1}}$$

The time unit has to be considered carefully in continuous processes as normally such processes operate continuously for only part of the total factory time. Usually there are three periods, start up, continuous processing (so-called steady state) and close down, and it is important to decide

what material balance is being studied. Also the time interval over which any measurements are taken must be long enough to allow for any slight periodic or chance variation.

In some instances a reaction takes place and the material balances have to be adjusted accordingly. Chemical changes can take place during a process, for example bacteria may be destroyed during heat processing, sugars may combine with amino acids, fats may be hydrolysed and these affect details of the material balance. The total mass of the system will remain the same but the constituent parts may change, for example in browning the sugars may reduce but browning compounds will increase. An example of the growth of microbial cells is given. Details of chemical and biological changes form a whole area for study in themselves, coming under the heading of unit processes or *reaction technology*.

EXAMPLE 2.6. Materials balance of yeast fermentation

Baker's yeast is to be grown in a continuous fermentation system using a fermenter volume of 20m³ in which the flow residence time is 16 h. A 2% inoculum containing 1.2 % of yeast cells is included in the growth medium. This is then passed to the fermenter, in which the yeast grows with a steady doubling time of 2.9h. The broth leaving the fermenter then passes to a continuous centrifuge, which produces a yeast cream containing 7% of yeast, 97% of the total yeast in the broth. Calculate the rate of flow of the yeast cream and of the residual broth from the centrifuge.

The volume of the fermenter is 20m³ and the residence time in this is 16 h so the flow rate through the fermenter must be:

$$20/16 = 1.25 \text{ m}^3\text{h}^{-1}$$

Assuming the broth to have a density substantially equal to that of water, i.e. 1000 kgm⁻³,

$$\text{Mass flow rate} = 1250\text{kg h}^{-1}$$

Yeast concentration in the liquid flowing to the fermenter

$$\begin{aligned} &= (\text{concentration in inoculum})/(\text{dilution of inoculum}) \\ &= (1.2/100)/(100/2) \\ &= 2.4 \times 10^{-4}\text{kgkg}^{-1} \end{aligned}$$

Now the yeast mass doubles every 2.9 h, so in 2.9h, 1kg becomes 1 x 2¹kg (1 generation)

$$\begin{aligned} \text{In 16h there are } 16/2.9 &= 5.52 \text{ doubling times} \\ 1\text{kg yeast grows to } 1 \times 2^{5.5}\text{kg} &= 45.9 \text{ kg} \\ \text{Yeast leaving fermenter} &= 2.4 \times 10^{-4} \times 45.9 \text{ kgkg}^{-1} \\ \text{Yeast leaving fermenter} &= \text{initial concentration} \times \text{growth} \times \text{flow rate} \\ &= 2.4 \times 10^{-4} \times 45.9 \times 1250 \\ &= 13.8 \text{ kgh}^{-1} \end{aligned}$$

From the centrifuge flows a (yeast rich) cream with 7% yeast, this being 97% of the total yeast:

$$\begin{aligned} \text{The yeast rich cream} &= (13.8 \times 0.97) \times 100/7 = 191 \text{ kgh}^{-1} \\ \text{and the broth (yeast lean) stream is} &= (1250 - 191) = 1059\text{kgh}^{-1} \\ \text{which contains } (13.8 \times 0.03) &= 0.41 \text{ kgh}^{-1} \text{ yeast} \\ \text{and the yeast concentration in the residual broth} &= 0.41/1059 \\ &= 0.039\% \end{aligned}$$

Materials balance over the centrifuge

Mass in	(kg h ⁻¹)	Mass out	(kg h ⁻¹)
Yeast-free broth	1236.2	Residual broth	1062
Yeast	13.8	(Yeast in broth 0.4)	
		Yeast cream	188
		(Yeast in cream 13.4)	
Total	1250.0	Total	1250.0

A materials balance, such as in Example 2.6 for the manufacture of yeast, could be prepared in much greater detail if this were necessary and if the appropriate information were available. Not only broad constituents, such as the yeast, can be balanced as indicated but all the other constituents must also balance.

One constituent is the element carbon: this comes with the yeast inoculum in the medium, which must have a suitable fermentable carbon source, for example it might be sucrose in molasses. The input carbon must then balance the output carbon, which will include the carbon in the outgoing yeast, carbon in the unused medium and also that which was converted to carbon dioxide and which came off as a gas or remained dissolved in the liquid. Similarly all of the other elements such as nitrogen and phosphorus can be balanced out and calculation of the balance can be used to determine what inputs are necessary knowing the final yeast production that is required and the expected yields. While a formal solution can be set out in terms of a number of simultaneous equations, it can often be easier both to visualize and to calculate if the data are tabulated and calculation proceeds step by step gradually filling out the whole detail.

Blending

Another class of situations that arises are blending problems in which various ingredients are combined in such proportions as to give a product of some desired composition. Complicated examples, in which an optimum or best achievable composition must be sought, need quite elaborate calculation methods, such as linear programming, but simple examples can be solved by straightforward mass balances.

EXAMPLE 2.7. Blending of minced meat

A processing plant is producing minced meat, which must contain 15% of fat. If this is to be made up from boneless cow beef with 23% of fat and from boneless bull beef with 5% of fat, what are the proportions in which these should be mixed?

Let the proportions be A of cow beef to B of bull beef.

Then by a mass balance on the fat,

$$\begin{aligned}A(0.23 - 0.15) &= B(0.15 - 0.05). \\A(0.08) &= B(0.10). \\A/B &= 10/8 \\ \text{or } A/(A + B) &= 10/18 \\ &= 5/9.\end{aligned}$$

i.e. 100kg of product will have 55.6 kg of cow beef to 44.4kg of bull beef.

It is possible to solve such a problem formally using algebraic equations and indeed all material balance problems are amenable to algebraic treatment. They reduce to sets of simultaneous equations and if the number of independent equations equals the number of unknowns the equations can be solved. For example, the blending problem above can be solved in this way.

If the weights of the constituents are A and B and proportions of fat are a , b , blended to give C of composition c :

$$\begin{array}{l} \text{then for fat} \\ \text{and overall} \end{array} \quad \begin{array}{l} Aa + Bb = Cc \\ A + B = C \end{array}$$

of which A and B are unknown, and say we require these to make up 100 kg of C then

$$\begin{array}{l} . \\ \text{or} \\ \text{and substituting into the first equation} \\ \text{or} \end{array} \quad \begin{array}{l} A + B = 100 \\ B = 100 - A \\ Aa + (100 - A)b = 100c \\ A(a - b) = 100(c - b) \\ A = 100 \frac{(c-b)}{(a-b)} \end{array}$$

and taking the numbers from the example

$$\begin{aligned} A &= 100 \frac{(0.15 - 0.05)}{(0.23 - 0.05)} \\ &= 100 \frac{0.10}{0.18} \\ &= 55.6\text{kg} \\ \text{and } B &= 44.4 \end{aligned}$$

as before, but the algebraic solution has really added nothing beyond a formula which could be useful if a number of blending operations were under consideration.

Layout

In setting up a material balance for a process a series of equations can be written for the various individual components and for the process as a whole. In some cases where groups of materials maintain constant ratios, then the equations can include such groups rather than their individual constituents. For example in drying vegetables, the carbohydrates, minerals, proteins etc., can be grouped together as 'dry solids', and then only dry solids and water need be taken through the material balance.

EXAMPLE 2.8. Drying yield of potatoes

Potatoes are dried from 14% total solids to 93% total solids. What is the product yield from each 1000 kg of raw potatoes assuming that 8% by weight of the original potatoes is lost in peeling.

Basis 1000kg potato entering

As 8% of potatoes are lost in peeling, potatoes to drying are 920 kg, solids 129 kg

Mass in (kg)		Mass out (kg)	
<i>Raw potatoes</i>		<i>Dried Product</i>	
Potato solids	140	Potato solids	129
Water	860	Associated water	<u>10</u>
		Total product	<u>139</u>
		<i>Losses</i>	
		Peelings	80
		solids	11
		water	69
		Water evaporated	<u>781</u>
		Total losses	<u>861</u>
Total	1000	Total	1000

$$\text{Product yield} = \frac{139 \times 100}{1000} = 14\%$$

Notice that numbers have been rounded to whole numbers, as this is appropriate accuracy.

Often it is important to be able to follow particular constituents of the raw material through a process. This is just a matter of calculating each constituent.

EXAMPLE 2.9. Extraction

1000 kg of soya beans, of composition 18% oil, 35% protein, 27.1% carbohydrate, 9.4% fibre and ash, 10.5% moisture, are:

- crushed and pressed, which reduces oil content in beans to 6%;
- then extracted with hexane to produce a meal containing 0.5% oil;
- finally dried to 8% moisture.

Assuming that there is no loss of protein and water with the oil, set out a materials balance for the soya bean constituents.

Basis 1000kg

Mass in:

$$\begin{aligned} \text{Oil} &= 1000 \times 18/100 = 180 \text{ kg} \\ \text{Protein} &= 1000 \times 35/100 = 350 \text{ kg} \\ \text{Other non-oil constituents} &= 470 \text{ kg} \end{aligned}$$

Carbohydrate, ash, fibre and water are calculated in a similar manner to fat and protein.

Mass out:

(a) Expressed oil.

In original beans, 820kg of protein, water, etc., are associated with 180 kg of oil.

In pressed material, 94 parts of protein, water, etc., are associated with 6 parts of oil.

$$\begin{aligned} \text{Total oil in expressed material} &= 820 \times 6/94 = 52.3 \text{ kg} \\ \text{Loss of oil in press} &= 180 - 52.3 = 127.7 \text{ kg} \end{aligned}$$

(b) Extracted oil.

In extracted meal 99.5 parts of protein, water, etc., are associated with 0.5 parts of oil.

$$\begin{aligned} \text{Total oil in extracted meal} &= 820 \times 0.5/99.5 = 4.1 \text{ kg} \\ \text{Loss of oil to hexane} &= 52.3 - 4.1 = 48.2 \text{ kg} \end{aligned}$$

(c) Water.

In the extracted meal, 8 parts of water are associated with 92 parts of oil, protein, etc.

$$\begin{aligned} \text{Weights of dry materials in final meal} &= 350 + 271 + 94 + 4.1 = 719.1 \text{ kg} \\ \text{Total water in dried meal} &= 719.1 \times 8/92 = 62.5 \text{ kg} \\ \text{Water loss in drying} &= 105 - 62.5 = 42.5 \text{ kg} \end{aligned}$$

MATERIALS BALANCE. BASIS 1000 kg SOYA BEANS ENTERING

Mass in (kg)		Mass out (kg)	
Oil	180	Expressed oil	127.7
Protein	350	Oil in hexane	<u>48.2</u>
Carbohydrate	271	Total oil	<u>175.9</u>
Ash and fibre	94	Total meal	<u>781.6</u>
Water	105	Consisting of:	
		Protein	350
		Carbohydrate	271
		Ash and fibre	94
		Water	62.5
		Oil	4.1
		<u>Water lost in drying</u>	<u>42.5</u>
<u>Total</u>	<u>1000</u>	<u>Total</u>	<u>1000.0</u>

ENERGY BALANCES

Energy takes many forms such as heat, kinetic energy, chemical energy, potential energy but because of interconversions it is not always easy to isolate separate constituents of energy balances. However, under some circumstances certain aspects predominate. In many heat balances, other forms of energy are insignificant; in some chemical situations, mechanical energy is insignificant and in some mechanical energy situations, as in the flow of fluids in pipes, the frictional losses appear as heat but the details of the heating need not be considered. We are seldom concerned with internal energies.

Therefore practical applications of energy balances tend to focus on particular dominant aspects and so a heat balance, for example, can be a useful description of important cost and quality aspects of a food process. When unfamiliar with the relative magnitudes of the various forms of energy entering into a particular processing situation, it is wise to put them all down. Then after some preliminary calculations, the important ones emerge and other minor ones can be lumped

together or even ignored without introducing substantial errors. With experience, the obviously minor ones can perhaps be left out completely though this always raises the possibility of error.

Energy balances can be calculated on the basis of external energy used per kilogram of product, or raw material processed, or on dry solids, or some key component. The energy consumed in food production includes:

direct energy which is fuel and electricity used on the farm, and in transport and in factories, and in storage, selling, etc.; and

indirect energy which is used to actually build the machines, to make the packaging, to produce the electricity and the oil and so on.

Food itself is a major energy source, and energy balances can be determined for animal or human feeding; food energy input can be balanced against outputs in heat and mechanical energy and chemical synthesis.

In the SI system there is only one energy unit, the joule. However, kilocalories are still used by some nutritionists, and British thermal units (Btu) in some heat-balance work.

The two applications used in this book are heat balances, which are the basis for heat transfer, and the energy balances used in analysing fluid flow.

Heat Balances

The most common important energy form is heat energy and the conservation of this can be illustrated by considering operations such as heating and drying. In these, enthalpy (total heat) is conserved. As with the material balances, so enthalpy balances can be written round the various items of equipment, or process stages, or round the whole plant, and it is assumed that no appreciable heat is converted to other forms of energy such as work.

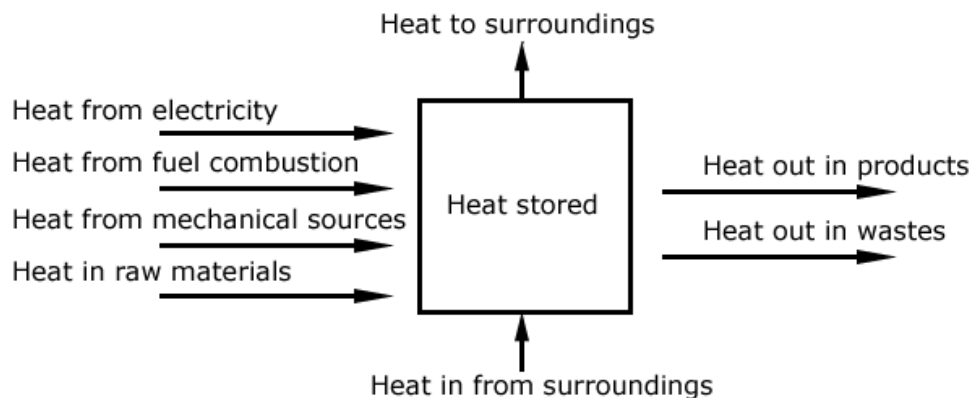


Figure 2.2. Heat balance.

Enthalpy (H) is always referred to some reference level or datum, so that the quantities are relative to this datum. Working out energy balances is then just a matter of considering the

various quantities of materials involved, their specific heats, and their changes in temperature or state (as quite frequently, latent heats arising from phase changes are encountered). Fig. 2.2 illustrates the heat balance.

Heat is absorbed or evolved by some reactions in food processing but usually the quantities are small when compared with the other forms of energy entering into food processing such as sensible heat and latent heat. Latent heat is the heat required to change, at constant temperature, the physical state of materials from solid to liquid, liquid to gas, or solid to gas. Sensible heat is the heat which when added or subtracted from food materials changes their temperature and thus can be sensed. The units of specific heat (c) are $\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ and sensible heat change is calculated by multiplying the mass by the specific heat and the change in temperature, $m c \Delta T$ and the unit is J. The unit of latent heat is J kg^{-1} and total latent heat change is calculated by multiplying the mass of the material, which changes its phase, by the latent heat. Having determined those factors that are significant in the overall energy balance, the simplified heat balance can then be used with confidence in industrial energy studies. Such calculations can be quite simple and straightforward but they give a quantitative feeling for the situation and can be of great use in design of equipment and process.

EXAMPLE 2.10. Heat demand in freezing bread.

It is desired to freeze 10,000 loaves of bread, each weighing 0.75 kg, from an initial room temperature of 18°C to a final store temperature of -18°C . If this is to be carried out in such a way that the maximum heat demand for the freezing is twice the average demand, estimate this maximum demand, if the total freezing time is to be 6 h.

If data on the actual bread is unavailable, in the literature are data on bread constituents, calculation methods and enthalpy/temperature tables.

(a) Tabulated data (Appendix 7) indicates specific heat above freezing $2.93 \text{ kJ kg}^{-1} \text{ } ^\circ\text{C}^{-1}$, below freezing $1.42 \text{ kJ kg}^{-1} \text{ } ^\circ\text{C}^{-1}$, latent heat of freezing 115 kJ kg^{-1} and freezing temperature is -2°C .

$$\text{Total enthalpy change } (\Delta H) = [18 - (-2)] 2.93 + 115 + [-2 - (-18)] 1.42 = 196 \text{ kJ kg}^{-1}.$$

(b) Formula (Appendix 7) assuming the bread is 36% water gives:

specific heat above freezing	$4.2 \times 0.36 + 0.84 \times 0.64$	$= 2.05 \text{ kJ kg}^{-1} \text{ } ^\circ\text{C}^{-1}$
specific heat below freezing	$2.1 \times 0.36 + 0.84 \times 0.64$	$= 1.29 \text{ kJ kg}^{-1} \text{ } ^\circ\text{C}^{-1}$
latent heat	0.36×335	$= 121 \text{ kJ kg}^{-1}$

$$\text{Total enthalpy change } (\Delta H) = [18 - (-2)] 2.04 + 121 + [-2 - (-18)] 1.29 = 183 \text{ kJ kg}^{-1}$$

(c) Enthalpy/temperature data for bread of 36% moisture (Mannheim *et al.*, 1957) suggest:

$$\begin{aligned} H_{18.3 \text{ } ^\circ\text{C}} &= 210.36 \text{ kJ kg}^{-1} \\ H_{-17.3 \text{ } ^\circ\text{C}} &= 65.35 \text{ kJ kg}^{-1} \end{aligned}$$

So from + 18°C to - 18°C total enthalpy change (ΔH) = 145kJkg⁻¹.

(d) The enthalpy/temperature data in Mannheim *et al.* 1957 can also be used to estimate "apparent" specific heats as $\Delta H / \Delta t = c$ and so using the data:

T °C	-20.6	-17.8	15.6	18.3
H kJkg ⁻¹	55.88	65.35	203.4	210.4

$$\text{Giving } c_{-18} = \frac{\Delta H}{\Delta t} = \frac{65.35 - 55.88}{20.6 - 17.8} = 3.4 \text{ kJkg}^{-1}\text{°C}^{-1},$$

$$\text{Giving } c_{18} = \frac{\Delta H}{\Delta t} = \frac{210.4 - 203.4}{18.3 - 15.6} = 2.6 \text{ kJkg}^{-1}\text{°C}^{-1}$$

Note that the "apparent" specific heat at -18°C, 3.4 kJkg⁻¹°C⁻¹, is higher than the specific heat below freezing in (a) 1.42 kJkg⁻¹°C⁻¹ and in (b) 1.29 kJkg⁻¹°C⁻¹. The reason for the high apparent specific heat at -18°C is due to some freezing still continuing at this temperature. It is suggested that at -18°C only about two-thirds of the water is actually frozen to ice. This implies only two-thirds of the latent heat has been extracted at this temperature. Making this adjustment to the latent-heat terms, estimates (a) and (b) give 158 kJkg⁻¹ and 142 kJkg⁻¹ respectively, much improving the agreement with (c) 145 kJkg⁻¹ for the total enthalpy change.

Taking ΔH as 150kJ kg⁻¹ then:

$$\begin{aligned} \text{Total heat change} &= 150 \times 10,000 \times 0.75 = 1.125 \times 10^6 \text{kJ} \\ \text{Total time} &= 6\text{h} = 2.16 \times 10^4 \text{s} \end{aligned}$$

$$(\Delta H / \Delta t) = 52 \text{ kJs}^{-1} = 52 \text{ kW on average.}$$

And if the maximum rate of heat removal is twice the average:

$$(\Delta H / \Delta t)_{\text{max}} = 2 \times 52 = 104 \text{ kW.}$$

Example 2.10 illustrates the application of heat balances, and it also illustrates the advisability of checking or obtaining corroborative data unless reliable experimental results are available for the particular system that is being considered. The straightforward application of the tabulated overall data would have produced a result about 30% higher than that finally calculated. On the other hand, for some engineering calculations to be within 30% may be as close as you can get.

In some cases, it is adequate to make approximations to heat balances by isolating dominant terms and ignoring less important ones. To make approximations with any confidence, it is necessary to be reasonably sure about the relative magnitudes of the quantities involved. Having once determined the factors that dominate the heat balance, simplified balances can then be set up if appropriate to the circumstances and used with confidence in industrial energy studies. This simplification reduces the calculation effort, focuses attention on the most important terms, and helps to inculcate in the engineer a quantitative feeling for the situation.

EXAMPLE 2.11. Dryer heat balance for casein drying.

In drying casein, the dryer is found to consume $4\text{m}^3/\text{h}$ of natural gas with a calorific value of $800\text{kJ}/\text{mole}$. If the throughput of the dryer is 60kg of wet casein per hour, drying it from 55% moisture to 10% moisture, estimate the overall thermal efficiency of the dryer taking into account the latent heat of evaporation only.

Basis: 1 hour of operation

60 kg of wet casein contains

$$\begin{aligned} 60 \times 0.55 \text{ kg water} &= 33\text{kg moisture} \\ \text{and } 60 \times (1-0.55) &= 27\text{kg bone dry casein.} \end{aligned}$$

As the final product contains 10% moisture, the moisture in the product is $27/9 = 3\text{kg}$

$$\begin{aligned} \text{So moisture removed} &= (33 - 3) = 30 \text{ kgh}^{-1} \\ \text{Latent heat of evaporation} &= 2257 \text{ kJ kg}^{-1} \text{ (at } 100^\circ\text{C from Appendix 8)} \\ \text{So heat necessary to supply} &= 30 \times 2257 \\ &= 6.8 \times 10^4 \text{ kJh}^{-1}. \end{aligned}$$

Assuming the natural gas to be at standard temperature and pressure at which 1 mole occupies 22.4 litres

$$\begin{aligned} \text{Rate of flow of natural gas} &= 4 \text{ m}^3 \text{ h}^{-1} = \frac{4 \times 1000}{22.4} = 179 \text{ moles h}^{-1} \\ \text{Heat available from combustion} &= 179 \times 800 = 14.3 \times 10^4 \text{ kJh}^{-1} \end{aligned}$$

$$\begin{aligned} \text{Approximate thermal efficiency of dryer} &= \frac{\text{heat needed}}{\text{heat used}} = 6.8 \times 10^4 / 14.3 \times 10^4 \\ &= \underline{48\%} \end{aligned}$$

To evaluate this efficiency more completely it would be necessary to take into account the sensible heat of the dry casein solids and the moisture, and the changes in temperature and humidity of the combustion air, which would be combined with the natural gas. However, as the latent heat of evaporation is the dominant term the above calculation gives a quick estimate and shows how a simple energy balance can give useful information.

Similarly energy balances can be carried out over thermal processing operations, and indeed any processing operations in which heat or other forms of energy are used.

EXAMPLE 2.12. Heat balance for cooling pea soup after canning

An autoclave contains 1000 cans of pea soup. It is heated to an overall temperature of 100°C . If the cans are to be cooled to 40°C before leaving the autoclave, how much cooling water is required if it enters at 15°C and leaves at 35°C ?

The specific heats of the pea soup and the can metal are respectively $4.1 \text{ kJ kg}^{-1}\text{ }^\circ\text{C}^{-1}$ and $0.50 \text{ kJ kg}^{-1} \text{ }^\circ\text{C}^{-1}$. The weight of each can is 60g and it contains 0.45 kg of pea soup. Assume that the heat content of the autoclave walls above 40°C is $1.6 \times 10^4 \text{ kJ}$ and that there is no heat loss through the walls.

Let w = the weight of cooling water required;

and the datum temperature be 40°C, the temperature of the cans leaving the autoclave.

Heat entering

Heat in cans = weight of cans x specific heat x temperature above datum
 = 1000 x 0.06 x 0.50 x (100-40)kJ
 = 1.8 x 10³kJ.

Heat in can contents = weight pea soup x specific heat x temperature above datum
 = 1000 x 0.45 x 4.1 x (100 - 40)kJ
 = 1.1 x 10⁵kJ.

Heat in water = weight of water x specific heat x temperature above datum
 = w x 4.21 x (15 - 40) (see Appendix 4 for specific heat of water)
 = - 105.3w kJ

Heat leaving

Heat in cans = 1000 x 0.06 x 0.50 x (40 - 40) (cans leave at datum temperature)
 = 0

Heat in can contents = 1000 x 0.45 x 4.1 x (40 - 40)
 = 0

Heat in water = w x 4.186 x (35 - 40)
 = -20.9 w

HEAT BALANCE OF COOLING PROCESS; 40°C AS DATUM LINE

Heat entering (kJ)		Heat Leaving (kJ)	
Heat in cans	1,800	Heat in cans	0
Heat in can contents	110,000	Heat in can contents	0
Heat in autoclave wall	16,000	Heat in autoclave walls	0
Heat in water	- 105.3w	Heat in water	-20.9w
Total heat entering	127.800 – 105.3w	Total heat leaving	-20.9w
	Total heat entering	= Total heat leaving	
	127,800 – 105.3w	= -20.9w	
	w	= 1514kg	

Amount of cooling water required, w = 1514 kg.

Other Forms of Energy

The most common mechanical power is motor power and it is usually derived, in food factories, from electrical energy but it can be produced from steam engines or waterpower. The electrical energy input can be measured by a suitable wattmeter, and the power used in the drive estimated. There are always losses from the motors due to heating, friction and windage; the motor

efficiency, which can normally be obtained from the motor manufacturer, expresses the proportion (usually as a percentage) of the electrical input energy which emerges usefully at the motor shaft and so is available.

When considering movement, whether of fluids in pumping, of solids in solids handling, or of foodstuffs in mixers, the energy input is largely mechanical. The flow situations can be analysed by recognising the conservation of total energy whether as energy of motion, or potential energy such as pressure energy, or energy lost in friction. Similarly, chemical energy released in combustion can be calculated from the heats of combustion of the fuels and their rates of consumption. Eventually energy emerges in the form of heat and its quantity can be estimated by summing the various sources.

EXAMPLE 2.13. Refrigeration load in bread freezing

The bread-freezing operation of Example 2.10 is to be carried out in an air-blast freezing tunnel. It is found that the fan motors are rated at a total of 80 horsepower and measurements suggest that they are operating at around 90% of their rating, under which conditions their manufacturer's data claims a motor efficiency of 86%. If 1 ton of refrigeration is 3.52 kW, estimate the maximum refrigeration load imposed by this freezing installation assuming (a) that fans and motors are all within the freezing tunnel insulation and (b) the fans but not their motors are in the tunnel. The heat-loss rate from the tunnel to the ambient air has been found to be 6.3 kW.

Extraction rate from freezing bread (maximum)	= 104 kW
Fan rated horsepower	= 80

Now 0.746 kW = 1 horsepower (Appendix 2) and the motor is operating at 90% of rating,

And so (fan + motor) power	= (80 x 0.9) x 0.746
	= 53.7 kW

(a) With motors + fans in tunnel	
heat load from fans + motors	= 53.7 kW
heat load from ambient	= 6.3 kW
Total heat load	= 104 + 53.7 + 6.3 kW
	= 164 kW
	= 164/3.52 (see Appendix 2)
	<u>= 46 tons of refrigeration</u>

(b) With motors outside, the motor inefficiency = (1 - 0.86) does not impose a load on the refrigeration.

Total heat load	= 104 + [0.86 x 53.7] + 6.3
	= 156 kW
	= 156/3.52
	<u>= 44.5 tons refrigeration</u>

In practice, material and energy balances are often combined, as the same stoichiometric information is needed for both.

SUMMARY

1. Material and energy balances can be worked out quantitatively knowing the amounts of materials entering into a process, and the nature of the process.
2. Material and energy balances take the basic form
Content of inputs = content of products + wastes/losses + changes in stored materials.
3. In continuous processes, a time balance must be established.
4. Energy includes heat energy (enthalpy), potential energy (energy of pressure or position), kinetic energy, work energy, chemical energy. It is the sum over all of these that is conserved.
5. Enthalpy balances, considering only heat, are useful in many food-processing situations.

PROBLEMS

1. If 5 kg of sucrose are dissolved in 20kg of water estimate the concentration of the solution in (a) w/w, (b) w/v, (c) mole fraction. (d) molal concentration. The density of a 20% sucrose solution is 1070kgm^{-3} , molecular weight of sucrose is 342.
(a) 20% (b) 21.4% (c) 0.018 (d) 0.63moles m^{-3}
2. 1 m^3 of air at a pressure of 1 atm is mixed with 0.1 m^3 of carbon dioxide at 1.5 atm and the mixture is compressed so that its total volume is 1 m^3 . Estimate the concentration of the carbon dioxide in the mixture in (a) w/w, (b) w/v, (c) mole fraction at a temperature of 25°C . Mean molecular weight of air is 28.8 and of carbon dioxide 44.
(a) 18.6% (b) 27% (c) 0.13
3. It is convenient to add salt to butter, produced in a continuous buttermaking machine, by adding a slurry of salt with water containing 60% of salt and 40% of water by weight. If the final composition of the butter is to be 15.8% moisture and 1.4% salt, estimate the original moisture content of the butter prior to salting.
(15.2%)
4. In a flour mill, wheat is to be adjusted to a moisture content of 15% on a dry basis. If the whole grain received at the mill is found to contain 11.4% of water initially, how much water must the miller add per 100kg of input grain as received, to produce the desired moisture content?
(1.8kg per 100kg)
5. (a) In an analysis, sugar beet is found to contain 75% of water and 17.5% of sugar. If of the remaining material, 25% is soluble and 75% insoluble, calculate the sugar content of the expressible juice assumed to contain water and all soluble solids pro rata.

(b) The beets are extracted by addition of a weight of water equal to their own weight and after a suitable period the soluble constituents are concentrated evenly throughout all the water present. Calculate the percentage of the total sugar left in the drained beet and the percentage of the total sugar extracted, assuming that the beet cells (insoluble) after the extraction have the same quantity of water associated with them as they did in the original beet.

(c) Lay out a materials balance for the extraction process

((a) 18.5% (b) drained beet 43%, extract 57%)

6. Sweet whey, following cheesemaking, has the following composition: 5.5% lactose, 0.8% protein, and 0.5% ash. The equilibrium solubility of lactose in water is:

Temp. °C	0	15	25	39	49	64
Lactose solubility kg/100 kg water	11.9	16.9	21.6	31.5	42.4	65.8

Calculate the percentage yield of lactose when 1000kg of whey is concentrated in a vacuum evaporator at 60°C to 60% solids and the concentrate is then cooled with crystallization of the lactose, down to 20°C over a period of weeks.

(83.6%)

7. In an ultrafiltration plant, whey is to be concentrated. Two streams are to be produced, a protein rich stream and a liquid stream (mainly water). In all 140,000 kg per day are to be processed to give a 12-fold concentration of 95% of the protein from an original whey concentration of 0.93% protein and 6% of other soluble solids.

Assuming that all of the soluble solids other than protein remain with the liquid stream, which also has 5% of the protein in it, estimate the daily flows and concentrations of the two product streams.

(protein stream: 11,084 kg day⁻¹, 11.16% protein)

(liquid stream: 128,916 kg day⁻¹, 0.05% protein)

8. It is desired to prepare a sweetened concentrated orange juice. The initial pressed juice contains 5 % of total solids and it is desired to lift this to 10% of total solids by evaporation and then to add sugar to give 2% of added sugar in the concentrated juice. Calculate the quantity of water that must be removed, and of sugar that must be added with respect to each 1000 kg of pressed juice.

(Water removed 500kg, sugar added 10.2kg)

9. A tomato-juice evaporator takes in juice at the rate of 1200kgh⁻¹. If the concentrated juice contains 35% of solids and the hourly rate of removal of water is 960 kg, calculate:

(a) % water in the original juice and

(b) the quantity of steam needed per hour for heating if the evaporator works at a pressure of 10kPa and the heat available from the steam is 2200kJkg⁻¹. Assume no heat losses.

((a) Water content of juice 93% (b) Steam needed 1089kgh⁻¹)

10. Processing water is to be heated in a direct fired heater, which burns natural gas with a calorific value of 20.2 MJ m^{-3} . If 5000 kg h^{-1} of this water has to be heated from 15°C to 80°C and the heater is estimated to be 45% efficient, estimate the hourly consumption of gas.
(150m^3)

11. In a casein factory (see Fig.2.3), the entering coagulum containing casein and lactose is passed through two cookers and acidified to remove the casein. The casein separates as a curd. The curd is removed from the whey by screening, and then washed, pressed and dried. The casein fines are removed from the pressed whey and the wash water by hydrocyclones, and mixed with the heated coagulum just before screening. The cycloned whey is used for heating in the first cooker and steam in the second cooker by indirect heating.

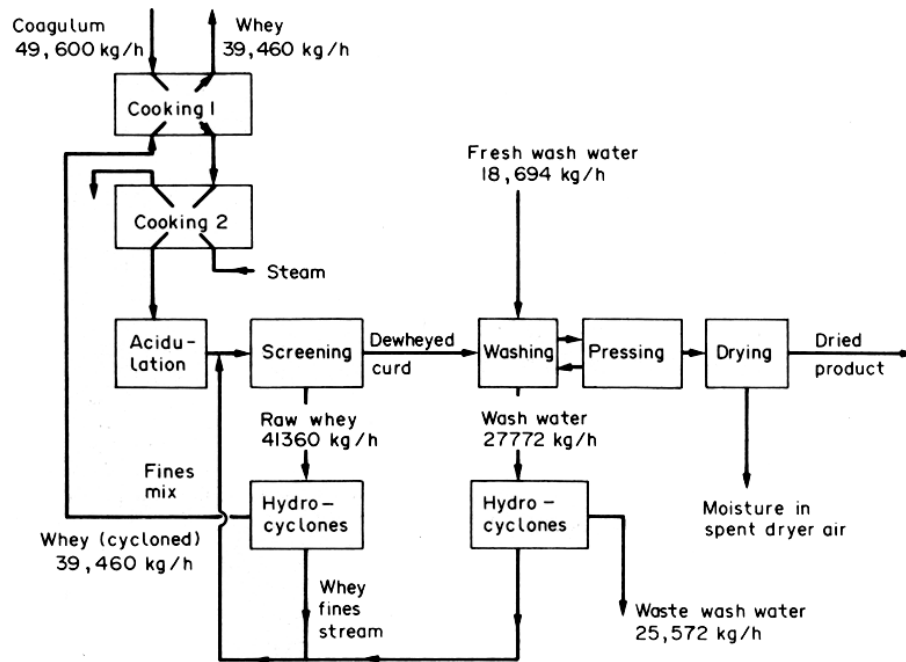


Figure 2.3 Casein Process

The casein and lactose contents of the various streams were determined

	%Composition on Wet Weight Basis	
	Casein	Lactose
Coagulum	2.76	3.68
Raw whey from screening	0.012	3.85
Whey (cycloned)	0.007	
Wash water	0.026	0.8
Waste wash water	0.008	

The moisture of the final dried product was 11.9%

From these data calculate a complete materials balance for the process, using a simple step-

by-step approach, starting with the hydrocyclones. Assume lactose completely soluble in all solutions and concentrations in fines and wastes streams from hydrocyclones are the same.

- (a) Set out an overall materials balance for the complete process to the production of the wet curd, i.e. until after the washing/pressing operation. Start the materials balance at the hydrocyclones, i.e. materials balances for the following operations: hydrocyclones, cooking, screening, washing; and then an overall balance.
- (b) Set out a casein mass balance over the same unit operations, and overall. Assume that there was little or no casein removed from the whey cycloned, and lost from coagulum.
- (c) Set out a lactose mass balance on the screening and washing.
- (d) Set out an overall materials balance from the wet curd to the dried product. Assume that the product contains only casein, lactose and water.
- (e) What was the composition of the wet curd (41.8% casein, 0.3% lactose, 57.9% water)
- (f) What was the composition of the final product (87.4% casein, 0.7% lactose, 11.9% water)
- (g) Determine the yield of the casein from the coagulum entering, and the losses in the cycloned whey and waste water. (yield 99.6%, loss in waste whey 0.2%, loss in waste water 0.2%)