CHAPTER 5

HEAT TRANSFER THEORY

Heat transfer is an operation that occurs repeatedly in the food industry. Whether it is called cooking, baking, drying, sterilizing or freezing, heat transfer is part of the processing of almost every food. An understanding of the principles that govern heat transfer is essential to an understanding of food processing.

Heat transfer is a dynamic process in which heat is transferred spontaneously from one body to another cooler body. The rate of heat transfer depends upon the differences in temperature between the bodies, the greater the difference in temperature, the greater the rate of heat transfer.

Temperature difference between the source of heat and the receiver of heat is therefore the driving force in heat transfer. An increase in the temperature difference increases the driving force and therefore increases the rate of heat transfer. The heat passing from one body to another travels through some medium which in general offers resistance to the heat flow. Both these factors, the temperature difference and the resistance to heat flow, affect the rate of heat transfer. As with other rate processes, these factors are connected by the general equation:

rate of transfer = driving force / resistance

For heat transfer:

rate of heat transfer = temperature difference/ heat flow resistance of medium

During processing, temperatures may change and therefore the rate of heat transfer will change. This is called *unsteady-state heat transfer*, in contrast to *steady-state heat transfer* when the temperatures do not change. An example of unsteady-state heat transfer is the heating and cooling of cans in a retort to sterilize the contents. Unsteady-state heat transfer is more complex since an additional variable, time, enters into the rate equations.

Heat can be transferred in three ways: by conduction, by radiation and by convection.

In conduction, the molecular energy is directly exchanged, from the hotter to the cooler regions, the molecules with greater energy communicating some of this energy to neighbouring molecules with less energy. An example of conduction is the heat transfer through the solid walls of a refrigerated store.

Radiation is the transfer of heat energy by electromagnetic waves, which transfer heat from one body to another, in the same way as electromagnetic light waves transfer light energy. An example of radiant heat transfer is when a foodstuff is passed below a bank of electric resistance heaters that are red-hot.

Convection is the transfer of heat by the movement of groups of molecules in a fluid. The groups of molecules may be moved by either density changes or by forced motion of the fluid. An example of convection heating is cooking in a jacketed pan: without a stirrer, density changes cause heat transfer by natural convection; with a stirrer, the convection is forced.

In general, heat is transferred in solids by conduction, in fluids by conduction and convection. Heat transfer by radiation occurs through open space, can often be neglected, and is most significant when temperature differences are substantial. In practice, the three types of heat transfer may occur together. For calculations it is often best to consider the mechanisms separately, and then to combine them where necessary.

HEAT CONDUCTION

In the case of heat conduction, the equation, rate = driving force/resistance, can be applied directly. The driving force is the temperature difference per unit length of heat-transfer path, also known as the temperature gradient. Instead of resistance to heat flow, its reciprocal called the conductance is used. This changes the form of the general equation to:

$$\text{rate of heat transfer} = \text{driving force x conductance},$$

that is:

$$dQ/dt = k\,A\,dT/dx \qquad (5.1)$$

where dQ/dt is the rate of heat transfer, the quantity of heat energy transferred per unit of time, A is the area of cross-section of the heat flow path, dT/dx is the temperature gradient, that is the rate of change of temperature per unit length of path, and k is the thermal conductivity of the medium. Notice the distinction between thermal conductance, which relates to the actual thickness of a given material (k/x) and thermal conductivity, which relates only to unit thickness.

The units of k, the thermal conductivity, can be found from eqn. (5.1) by transposing the terms

$$k = dQ/dt \text{ x } 1/A \text{ x } 1/(dT/dx)$$

$$= Js^{-1} \text{ x } m^{-2} \text{ x } 1/({}^{o}C\ m^{-1})$$
$$= Jm^{-1}s^{-1}\,{}^{o}C^{-1}$$

Equation (5.1) is known as the Fourier equation for heat conduction.

Note: Heat flows from a hotter to a colder body that is in the direction of the negative temperature gradient. Thus a minus sign should appear in the Fourier equation. However, in simple problems the direction of heat flow is obvious and the minus sign is considered to be confusing rather than helpful, so it has not been used.

Thermal Conductivity

On the basis of eqn. (5.1) thermal conductivities of materials can be measured. Thermal conductivity does change slightly with temperature, but in many applications it can be regarded as a constant for a given material. Thermal conductivities are given in Appendices 3,4,5,6, which give physical properties of many materials used in the food industry.

In general, metals have a high thermal conductivity, in the region 50-400 $Jm^{-1}s^{-1}\ {}^oC^{-1}$. Most foodstuffs contain a high proportion of water and as the thermal conductivity of water is about 0.7 $Jm^{-1}s^{-1}\ {}^oC^{-1}$ above 0^oC, thermal conductivities of foods are in the range 0.6 – 0.7 $Jm^{-1}s^{-1}\ {}^oC^{-1}$. Ice has a substantially higher thermal conductivity than water, about 2.3 $Jm^{-1}s^{-1}\ {}^oC^{-1}$. The thermal conductivity of frozen foods is, therefore, higher than foods at normal temperatures.

Most dense non-metallic materials have thermal conductivities of 0.5-2 $Jm^{-1}s^{-1}\ {}^oC^{-1}$. Insulating materials, such as those used in walls of cold stores, approximate closely to the conductivity of gases as they are made from non-metallic materials enclosing small bubbles of gas or air. The conductivity of air is 0.024 $Jm^{-1}s^{-1}\ {}^oC^{-1}$ at 0^oC, and insulating materials such as foamed plastics, cork and expanded rubber are in the range 0.03 - 0.06 $Jm^{-1}s^{-1}\ {}^oC^{-1}$. Some of the new in foamed plastic materials have thermal conductivities as low as 0.026 $Jm^{-1}s^{-1}\ {}^oC^{-1}$.

When using published tables of data, the units should be carefully checked. Mixed units, convenient for particular applications, are sometimes used and they may need to be converted.

Conduction through a Slab

If a slab of material, as shown in Fig. 5.1, has two faces at different temperatures T_1 and T_2 heat will flow from the face at the higher temperature T_1 to the other face at the lower temperature T_2.

The rate of heat transfer is given by Fourier's equation:

$$dQ/dt = kA\, dT/dx$$

Under steady temperature conditions dQ/dt = constant, which may be called q:

and so $$q = kA\, dT/dx$$

but dT/dx, the rate of change of temperature per unit length of path, is given by $(T_1 - T_2)/x$ where x is the thickness of the slab,

so $$q = kA(T_1 - T_2)/x$$

or $$q = kA\, \Delta T/x$$

$$= (k/x)\, A\, \Delta T \qquad (5.2)$$

This may be regarded as the basic equation for simple heat conduction. It can be used to calculate the rate of heat transfer through a uniform wall if the temperature difference across it and the thermal conductivity of the wall material are known.

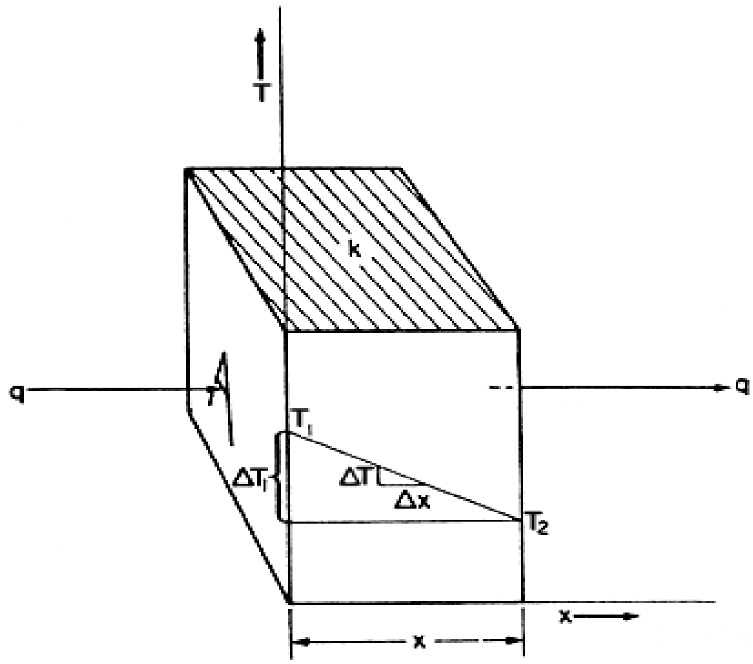

Figure 5.1. Heat conduction through a slab

EXAMPLE 5.1. Rate of heat transfer in cork

A cork slab 10cm thick has one face at – 12°C and the other face at 21°C. If the mean thermal conductivity of cork in this temperature range is 0.042 $Jm^{-1}s^{-1}\ {}^{o}C^{-1}$, what is the rate of heat transfer through 1 m^2 of wall?

$T_1 = 21^oC$ $\quad T_2 = -12^oC$ $\quad \Delta T = 33^oC$

$A = 1m^2$ $\quad k = 0.042\ Jm^{-1}s^{-1}\ {}^oC^{-1}$ $\quad x = 0.l\ m$

$$q = \frac{0.042}{0.1} \times 1 \times 33$$

$$= \underline{13.9\ Js^{-1}}$$

Heat Conductances

In tables of properties of insulating materials, heat conductances are sometimes used instead of thermal conductivities. The heat conductance is the quantity of heat that will pass in unit time, through unit area of a specified thickness of material, under unit temperature difference. For a thickness x of material with a thermal conductivity of k in $Jm^{-1}s^{-1}\ {}^oC^{-1}$, the conductance is $k/x = C$ and the units of conductance are $Jm^{-2}s^{-1}\ {}^oC^{-1}$.

Heat conductance = $C = k/x$.

Heat Conductances in Series

Frequently in heat conduction, heat passes through several consecutive layers of different materials. For example, in a cold store wall, heat might pass through brick, plaster, wood and cork.

In this case, eqn. (5.2) can be applied to each layer. This is illustrated in Fig. 5.2.

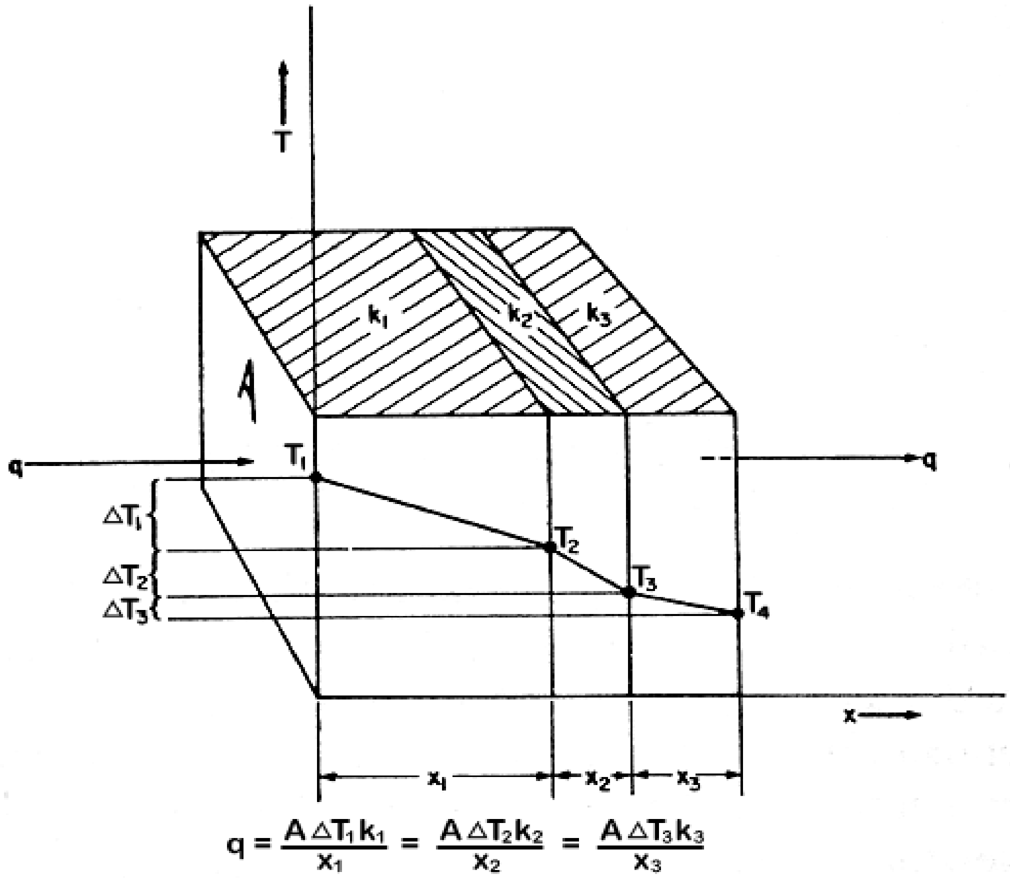

Figure 5.2 Heat conductances in series

In the steady state, the same quantity of heat per unit time must pass through each layer.

$$q = A_1 \Delta T_1 \, k_1/x_1 = A_2 \Delta T_2 \, k_2/x_2 = A_3 \, \Delta T_3 \, k_3/x_3 = \ldots\ldots$$

If the areas are the same,

$$A_1 = A_2 = A_3 = \ldots\ldots = A$$

$$q = A \Delta T_1 \, k_1/x_1 = A \, \Delta T_2 \, k_2/x_2 = A \, \Delta T_3 \, k_3/x_3 = \ldots\ldots$$

So $A\Delta T_1 = q(x_1/k_1)$ and $A\Delta T_2 = q(x_2/k_2)$ and $A\Delta T_3 = q(x_3/k_3)$……

$$A\Delta T_1 + A\Delta T_2 + A\Delta T_3 + \ldots = q(x_1/k_1) + q(x_2/k_2) + q(x_3/k_3) + \ldots$$

$$A(\Delta T_1 + \Delta T_2 + \Delta T_3 + ..) = q(x_1/k_1 + x_2/k_2 + x_3/k_3 + \ldots)$$

The sum of the temperature differences over each layer is equal to the difference in temperature of the two outside surfaces of the complete system, i.e.

$$\Delta T_1 + \Delta T_2 + \Delta T_3 + \ldots = \Delta T$$

and since k_1/x_1 is equal to the conductance of the material in the first layer, C_1, and k_2/x_2 is equal to the conductance of the material in the second layer C_2,

$$x_1/k_1 + x_2/k_2 + x_3/k_3 + \ldots = 1/C_1 + 1/C_2 + 1/C_3 \ldots\ldots$$
$$= 1/U$$

where U = the overall conductance for the combined layers, in $Jm^{-2}s^{-1}\,{}^{o}C^{-1}$.

Therefore $A\,\Delta T = q\,(1/U)$

And so $q = UA\,\Delta T$ (5.3)

This is of the same form as eqn (5.2) but extended to cover the composite slab. U is called the overall heat-transfer coefficient, as it can also include combinations involving the other methods of heat transfer – convection and radiation.

EXAMPLE 5.2. Heat transfer in cold store wall of brick, concrete and cork

A cold store has a wall comprising 11 cm of brick on the outside, then 7.5 cm of concrete and then 10cm of cork. The mean temperature within the store is maintained at –18°C and the mean temperature of the outside surface of the wall is 18°C.

Calculate the rate of heat transfer through $1m^2$ of wall. The appropriate thermal conductivities are for brick, concrete and cork, respectively 0.69, 0.76 and 0.043 $Jm^{-1}s^{-1}\,{}^{o}C^{-1}$.

Determine also the temperature at the interface between the concrete and the cork layers.

For brick x_1/k_1 = 0.11/0.69 = 0.16.
For concrete x_2/k_2 = 0.075/0.76 = 0.10.
For cork x_3/k_3 = 0.10/0.043 = 2.33

.

But $1/U = x_1/k_1 + x_2/k_2 + x_3/k_3$
$= 0.16 + 0.10 + 2.33$
$= 2.59$

Therefore $U = 0.38\ Jm^{-2}s^{-1}\,{}^{o}C^{-1}$
$\Delta T = 18 - (-18) = 36^{o}C$

$A \quad = 1m^2$

$$q = UA\,\Delta T$$
$$= 0.38 \times 1 \times 36$$
$$= \underline{13.7 Js^{-1}}$$

Further, $q = A_3\,\Delta T_3 k_3/x_3$

and for the cork wall $A_3 = 1\ m^2$, $x_3/k_3 = 2.33$ and $q = 13.7 Js^{-1}$

Therefore $13.7 = 1 \times \Delta T_3 \times 1/2.33$ from rearranging eqn. (5.2)

$\Delta T_3 = 32^oC$.

But ΔT_3 is the difference between the temperature of the cork/concrete surface and the temperature of the cork surface inside the cold store.

Therefore $T_c - (-18) = 32$

where T_c is the temperature at the cork/concrete surface

and so $T_c = \underline{14^oC}$.

If ΔT_1 is the difference between the temperature of the brick/concrete surface T_b and the temperature of the external air

Then $13.7 = 1 \times \Delta T_1 \times 1/0.16 = 6.25\,\Delta T_1$

Therefore $18 - T_b = \Delta T_1 = 13.7/6.25 = 2.2$

and so $\underline{T_b = 15.8^oC}$

Working it through shows approximate interface temperatures: air/brick 18°C, brick/concrete 16°C, concrete/cork 14°C, and cork/air -18°C. This shows that almost all of the temperature difference occurs across the insulation (cork): the actual temperatures can be significant especially if they lie below the temperature at which the atmospheric air condenses, or freezes.

Heat Conductances in Parallel

Heat conductances in parallel have a sandwich construction at right angles to the direction of the heat transfer, but with heat conductances in parallel, the material surfaces are parallel to the direction of heat transfer and to each other. The heat is therefore passing through each material at the same time, instead of through one material and then the next. This is illustrated in Fig. 5.3.

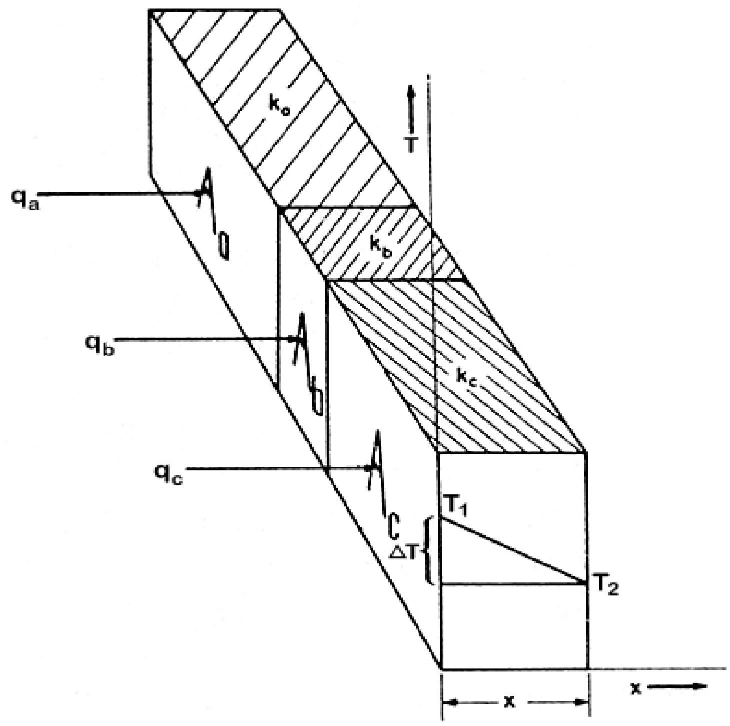

Figure 5.3. Heat conductances in parallel

An example is the insulated wall of a refrigerator or an oven, in which the walls are held together by bolts. The bolts are in parallel with the direction of the heat transfer through the wall: they carry most of the heat transferred and thus account for most of the losses.

EXAMPLE 5.3. Heat transfer in walls of a bakery oven

The wall of a bakery oven is built of insulating brick 10 cm thick and of thermal conductivity 0.22 $Jm^{-1}s^{-1o}C^{-1}$. Steel reinforcing members penetrate the brick, and their total area of cross-section represents 1% of the inside wall area of the oven.

If the thermal conductivity of the steel is 45 $Jm^{-1}s^{-1o}C^{-1}$, calculate (a) the relative proportions of the total heat transferred through the wall by the brick and by the steel and (b) the heat loss for each m^2 of oven wall if the inner side of the wall is at 230°C and the outer side is at 25°C.

Applying eqn. (5.1) $q = A\,\Delta T\,k/x$, we know that ΔT is the same for the bricks and for the steel. Also x, the thickness, is the same.

(a) Consider the loss through an area of 1 m^2 of wall (0.99m^2 of brick, and 0.01 m^2 of steel)

For brick $q_b = A_b\,\Delta T\,k_b/x$

$$= \frac{0.99(230 - 25)0.22}{0.10}$$

$$= 446 Js^{-1}$$

For steel q_s $= A_s \Delta T\, k_s/x$

$= \dfrac{\underline{0.01(230 - 25)45}}{0.10}$

$= 923 Js^{-1}$

Therefore q_b / q_s $= \underline{0.48}$

(b)Total heat loss

q $= (q_b + q_s)$ per m^2 of wall

$= 446 + 923$

$= \underline{1369 Js^{-1}}$

Therefore percentage of heat carried by steel

$= (923/1369) \times 100$

$= \underline{67\%}$

SURFACE HEAT TRANSFER

Newton found, experimentally, that the rate of cooling of the surface of a solid, immersed in a colder fluid, was proportional to the difference between the temperature of the surface of the solid and the temperature of the cooling fluid. This is known as Newton's Law of Cooling, and it can be expressed by the equation, analogous to eqn. (5.2),

$$q = h_s A\,(T_a - T_s) \qquad (5.4)$$

where h_s is called the surface heat-transfer coefficient, T_a is the temperature of the cooling fluid and T_s is the temperature at the surface of the solid. The surface heat-transfer coefficient can be regarded as the conductance of a hypothetical surface film of the cooling medium of thickness x_f such that $h_s = k_f / x_f$, where k_f is the thermal conductivity of the cooling medium.

Following on this reasoning, it may be seen that h_s can be considered as arising from the presence of another layer, this time at the surface, added to the case of the composite slab considered previously. The heat passes through the surface, then through the various elements of a composite slab and then it may pass through a further surface film. We can at once write the important equation:

$$q = A\Delta T[(1/h_{s1}) + x_1/k_1 + x_2/k_2 + .. \quad + (1/h_{s2})] \qquad (5.5)$$

$$= UA\,\Delta T$$

where $1/U = (1/h_{s1}) + x_1/k_1 + x_2/k_2 + .. \quad + (1/h_{s2})$

and h_{s1}, $h_{s1}$are the surface coefficients on either side of the composite slab, x_1, x_2...... are the thicknesses of the layers making up the slab, and k_1, k_2... are the conductivities of layers of thickness x_1,..... . The coefficient h_s is also known as the convection heat-transfer coefficient and values for it will be discussed in detail under the heading of convection. It is useful at this point, however, to appreciate the magnitude of h_s under various common conditions and these are shown in Table 5.1.

TABLE 5.1
APPROXIMATE RANGE OF SURFACE HEAT-TRANSFER COEFFICIENTS

	h $Jm^{-2}s^{-1}\ {}^oC^{-1}$
Boiling liquids	2400-24,000
Condensing liquids	1800-18,000
Still air	6
Moving air ($3ms^{-1}$)	30
Liquids flowing through pipes	1200 - 6000

EXAMPLE 5.4. Heat transfer in jacketed pan
Sugar solution is being heated in a jacketed pan made from stainless steel, 1.6 mm thick. Heat is supplied by condensing steam at 200kPa gauge in the jacket. The surface transfer coefficients are, for condensing steam and for the sugar solution, 12,000 and 3000 $Jm^{-2}s^{-1}{}^oC^{-1}$ respectively, and the thermal conductivity of stainless steel is 21 $Jm^{-1}s^{-1}{}^oC^{-1}$.
Calculate the quantity of steam being condensed per minute if the transfer surface is 1.4 m^2 and the temperature of the sugar solution is 83^oC.

From steam tables, Appendix 8, the saturation temperature of steam at 200 kPa gauge(300kPa absolute) $=134^oC$ and the latent heat = $2164kJkg^{-1}$.

For stainless steel $x/k = 0.0016/21 = 7.6 \times 10^{-5}$

ΔT = (condensing temperature of steam) - (temperature of sugar solution)
= $134 - 83 = 51^oC$.

From eqn. (5.5)

$1/U = 1/12{,}000 + 7.6 \times 10^{-5} + 1/3000$
$U = 2032\ Jm^{-2}s^{-1}{}^oC^{-1}$

and since $A = 1.4m^2$

$q = UA\,\Delta T$
$= 2032 \times 1.4 \times 51$
$= 1.45 \times 10^5\ Js^{-1}$

Therefore steam required = heat transferred per sec /latent heat from steam
$= 1.45 \times 10^5 / (2.164 \times 10^6)\ kgs^{-1}$

$= 0.067 kgs^{-1}$
$= \underline{4\ kg\ min^{-1}}$

UNSTEADY-STATE HEAT TRANSFER

In food-process engineering, heat transfer is very often in the unsteady state, in which temperatures are changing and materials are warming or cooling. Unfortunately, study of heat flow under these conditions is complicated. In fact, it is the subject for study in a substantial branch of applied mathematics, involving finding solutions for the Fourier equation written in terms of partial differentials in three dimensions. There are some cases that can be simplified and handled by elementary methods, and also charts have been prepared which can be used to obtain numerical solutions under some conditions of practical importance.

A simple case of unsteady-state heat transfer arises from the heating or cooling of solid bodies made from good thermal conductors, for example a long cylinder, such as a meat sausage or a metal bar, being cooled in air. The rate at which heat is being transferred to the air from the surface of the cylinder is given by eqn. (5.4)

$$q = dQ/dt = h_s A(T_s - T_a)$$

where T_a is the air temperature and T_s is the surface temperature.

Now, the heat being lost from the surface must be transferred to the surface from the interior of the cylinder by conduction. This heat transfer from the interior to the surface is difficult to determine but as an approximation, we can consider that all the heat is being transferred from the centre of the cylinder. In this instance, we evaluate the temperature drop required to produce the same rate of heat flow from the centre to the surface as passes from the surface to the air. This requires a greater temperature drop than the actual case in which much of the heat has in fact a shorter path.

Assuming that all the heat flows from the centre of the cylinder to the outside, we can write the conduction equation

$$dQ/dt = (k/L)A(T_c - T_s)$$

where T_c is the temperature at the centre of the cylinder, k is the thermal conductivity of the material of the cylinder and L is the <u>radius</u> of the cylinder.

Equating these rates:

$$h_s A(T_s - T_a) = (k/L)A(T_c - T_s)$$

$$h_s(T_s - T_a) = (k/L)(T_c - T_s)$$

and so

$$h_s L/k = (T_c - T_s)/(T_s - T_a)$$

To take a practical case of a copper cylinder of 15cm radius cooling in air, k_c = 380 $Jm^{-1}s^{-1}\,^oC^{-1}$, h_s = 30 $Jm^{-2}s^{-1}\,^oC^{-1}$ (from Table 5.1), L = 0.15m,

$$(T_c - T_s)/\,(T_s - T_a) = (30 \times 0.15)/380$$
$$= 0.012$$

In this case 99% of the temperature drop occurs between the air and the cylinder surface. By comparison with the temperature drop between the surface of the cylinder and the air, the temperature drop within the cylinder can be neglected. On the other hand, if the cylinder were made of a poorer conductor as in the case of the sausage, or if it were very large in diameter, or if the surface heat-transfer coefficient were very much larger, the internal temperature drops could not be neglected.

This simple analysis shows the importance of the ratio:

$$\frac{\text{heat transfer coefficient at the surface}}{\text{heat conductance to the centre of the solid}} = h_s L/k$$

This dimensionless ratio is called the Biot number (Bi) and it is important when considering unsteady state heat flow. When (Bi) is small, and for practical purposes this may be taken as any value less than about 0.2, the interior of the solid and its surface may be considered to be all at one uniform temperature. In the case in which (Bi) is less than 0.2, a simple analysis can be used, therefore, to predict the rate of cooling of a solid body.

Therefore for a cylinder of a good conductor, being cooled in air,

$$dQ = h_s A(T_s - T_a)\,dt$$

But this loss of heat cools the cylinder in accordance with the usual specific heat equation:

$$dQ = c\rho V dT$$

where c is the specific heat of the material of the cylinder, ρ is the density of this material and V is the volume of the cylinder.

Since the heat passing through the surface must equal the heat lost from the cylinder, these two expressions for dQ can be equated:

$$c\rho V dT = h_s A(T_s - T_a)\,dt$$

Integrating between $T_s = T_1$ and $T_s = T_2$, the initial and final temperatures of the cylinder during the cooling period, t , we have:

$$- h_s At/c\rho V = \log_e (T_2 - T_a)/(T_1 - T_a)$$

or

$$(T_2 - T_a)/(T_1 - T_a) = \exp(\,-h_s At/c\rho V\,) \qquad (5.6)$$

For this case, the temperatures for any desired interval can be calculated, if the surface transfer coefficient and the other physical factors are known. This gives a reasonable approximation so long as (Bi) is less than about 0.2. Where (Bi) is greater than 0.2 the centre of the solid will cool more slowly than this equation suggests. The equation is not restricted to cylinders, it applies to solids of any shape so long as the restriction in (Bi), calculated for the smallest half-dimension, is obeyed.

Charts have been prepared which give the temperature relationships for solids of simple shapes under more general conditions of unsteady-state conduction. These charts have been calculated from solutions of the conduction equation and they are plotted in terms of dimensionless groups so that their application is more general. The form of the solution is:

$$f\{(T - T_0)/(T_i - T_0)\} = F\{(kt/c\rho L^2)(h_s L/k)\} \qquad (5.7)$$

where f and F indicate functions of the terms following, T_i is the initial temperature of the solid, T_0 is the temperature of the cooling or heating medium, T is the temperature of the solid at time t, $(kt/c\rho L^2)$ is called the Fourier number (Fo) (this includes the factor $k/c\rho$ the thermal conductance divided by the volumetric heat capacity, which is called the thermal diffusivity) and $(h_s L/k)$ is the Biot number.

A mathematical outcome that is very useful in these calculations connects results for two- and three-dimensional situations with results from one-dimensional situations. This states that the two- and three-dimensional values called $F(x,y)$ and $F(x,y,z)$ can be obtained from the individual one-dimensional results if these are $F(x)$, $F(y)$ and $F(z)$, by simple multiplication:

$$F(x,y) = F(x)F(y)$$

and

$$F(x,y,z) = F(x)F(y)F(z)$$

Using the above result, the solution for the cooling or heating of a brick is obtained from the product of three slab solutions. The solution for a cylinder of finite length, such as a can, is obtained from the product of the solution for an infinite cylinder, accounting for the sides of the can, and the solution for a slab, accounting for the ends of the can.

Charts giving rates of unsteady-state heat transfer to the centre of a slab, a cylinder, or a sphere, are given in Fig.5.4. On one axis is plotted the fractional unaccomplished temperature change, $(T - T_0)/(T_i - T_0)$. On the other axis is the Fourier number $(kt/c\rho L^2)$, which may be thought of in this connection as a time coordinate. The various curves are for different values of the reciprocal of the Biot number, k/hr for spheres, k/hL for slabs. More detailed charts, giving surface and mean temperatures in addition to centre temperatures, may be found in McAdams (1954), Fishenden and Saunders (1950) and Perry (1997).

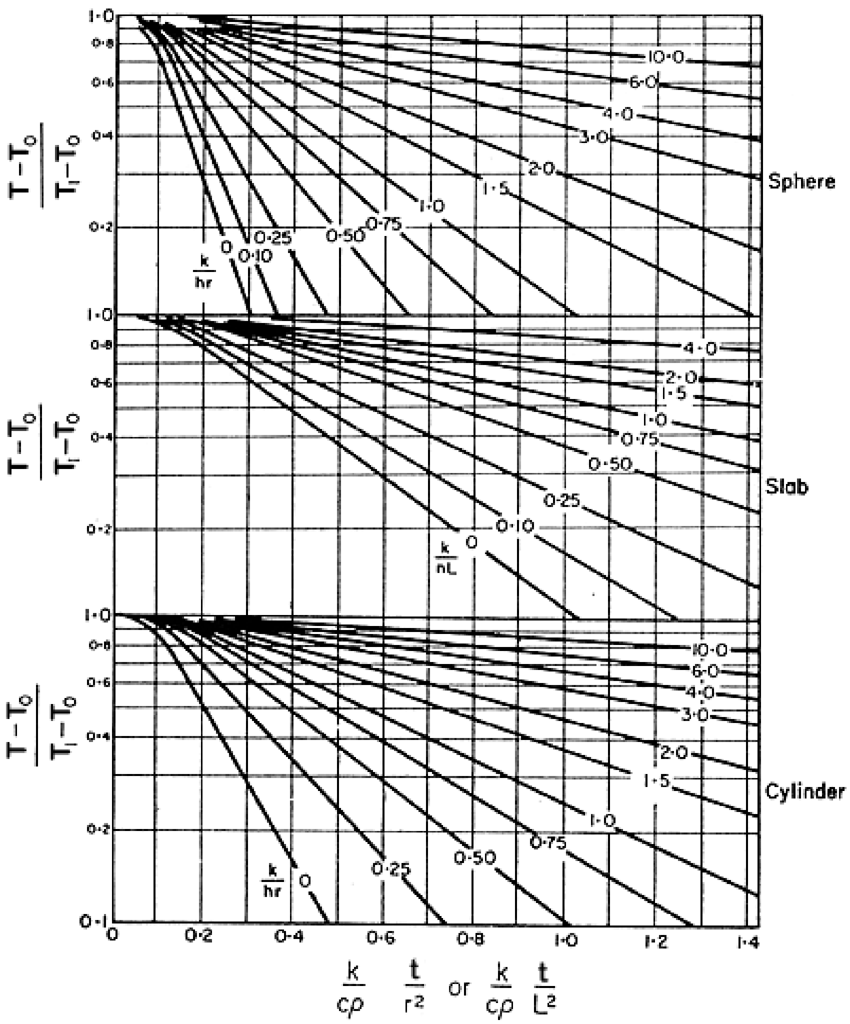

Figure 5.4. Transient heat conduction
Temperatures at the centre of a sphere, slab and cylinder adapted from Henderson and Perry, *Agricultural Process Engineering*, 1955

EXAMPLE 5.5. Heat transfer in cooking sausages
A process is under consideration in which large cylindrical meat sausages are to be processed in an autoclave. The sausage may be taken as thermally equivalent to a cylinder 30cm long and 10cm in diameter. If the sausages are initially at a temperature of 21°C and the temperature in the autoclave is maintained at 116°C, estimate the temperature of the sausage at its centre 2h after it has been placed in the autoclave.
Assume that the thermal conductivity of the sausage is 0.48 $Jm^{-1}s^{-1}{}^{o}C^{-1}$**,** that its specific gravity is 1.07, and its specific heat is 3350 $Jkg^{-1}{}^{o}C^{-1}$. The surface heat-transfer coefficient in the autoclave to the surface of the sausage is l200$Jm^{-2}s^{-1}\,{}^{o}C^{-1}$.

This problem can be solved by combining the unsteady-state solutions for a cylinder with those for a slab, working from Fig. 5.4.

(a) For the cylinder, of radius r = 5 cm (instead of L in this case)

$$\text{Bi} = h_s r/k = (1200 \times 0.05)/0.48 = 125$$

(Often in these systems, the length dimension used as parameter in the charts is the half-thickness, or the radius, but this has to be checked on the graphs used.)

So $1/(\text{Bi}) = 8 \times 10^{-3}$

After 2 hours $t = 7200\text{s}$

Therefore $\text{Fo} = kt/c\rho r^2 = (0.48 \times 7200)/[3350 \times 1.07 \times 1000 \times (0.05)^2]$

$= 0.39$

and so from Fig. 5.3 for the cylinder:

$$(T - T_0)/(T_i - T_0) = 0.175 = \text{say, } F(x).$$

(b) For the slab the half-thickness 30/2 cm = 0.15m

and so $\text{Bi} = h_s L/k = (1200 \times 0.15)/0.48 = 375$

$1/\text{Bi} = 2.7 \times 10^{-3}$

$t = 7200\text{s}$ as before and

$$kt/c\rho L^2 = (0.48 \times 7200)/[3350 \times 1.07 \times 1000 \times (0.15)^2]$$

$$= 4.3 \times 10^{-2}$$

and so from Fig. 5.3 for the slab:

$$(T - T_0)/(T_i - T_0) = 0.98 = \text{say, } F(y)$$

So overall $(T - T_0)/(T_i - T_0) = F(x)\,F(y)$

$= 0.175 \times 0.98$

$= 0.172$

$T_i = 21°\text{C}$ and $T_0 = 116°\text{C}$, and T_2 is the temperature at the centre

Therefore $$\frac{T_2 - 116}{21 - 116} = 0.172$$

Therefore T_2 = $\underline{100°\text{C}}$

RADIATION HEAT TRANSFER

Radiation heat transfer is the transfer of heat energy by electromagnetic radiation. Radiation operates independently of the medium through which it occurs and depends upon the relative temperatures, geometric arrangements and surface structures of the materials that are emitting or absorbing heat.

The calculation of radiant heat transfer rates, in detail, is beyond the scope of this book and for most food processing operations a simplified treatment is sufficient to estimate radiant heat effects. Radiation can be significant with small temperature differences as, for example, in freeze drying and in cold stores, but it is generally more important where the temperature differences are greater. Under these circumstances, it is often the most significant mode of heat transfer, for example in bakers' ovens and in radiant dryers.

The basic formula for radiant-heat transfer is the Stefan-Boltzmann Law

$$q = A \sigma \boldsymbol{T}^4 \qquad (5.8)$$

where ***T*** is the absolute temperature (measured from the absolute zero of temperature at – 273°C, and indicted in Bold type) in degrees Kelvin (K) in the SI system, and σ (sigma) is the Stefan-Boltzmann constant = 5.73 x 10^{-8} J $m^{-2}s^{-1}K^{-4}$. The absolute temperatures are calculated by the formula K = (°C + 273).

This law gives the radiation emitted by a perfect radiator (a black body, as this is called, though it could be a red-hot wire in actuality). A black body gives the maximum amount of emitted radiation possible at its particular temperature. Real surfaces at a temperature ***T*** do not emit as much energy as predicted by eqn. (5.8), but it has been found that many emit a constant fraction of it. For these real bodies, including foods and equipment surfaces, that emit a constant fraction of the radiation from a black body, the equation can be rewritten

$$q = \varepsilon A \sigma \boldsymbol{T}^4 \qquad (5.9)$$

where ε (epsilon) is called the emissivity of the particular body and is a number between 0 and 1. Bodies obeying this equation are called grey bodies.

Emissivities vary with the temperature ***T*** and with the wavelength of the radiation emitted. For many purposes, it is sufficient to assume that for:

*dull black surfaces (lamp-black or burnt toast, for example), emissivity is approximately 1;
*surfaces such as paper/painted metal/wood and including most foods, emissivities are about 0.9;
*rough unpolished metal surfaces, emissivities vary from 0.7 to 0.25;
*polished metal surfaces, emissivities are about or below 0.05.

These values apply at the low and moderate temperatures, which are those encountered in food processing.

Just as a black body emits radiation, it also absorbs it and according to the same law, eqn. (5.8). Again grey bodies absorb a fraction of the quantity that a black body would absorb, corresponding this time to their absorptivity α (alpha). For grey bodies it can be shown that α = ε. The fraction of the incident radiation that is not absorbed is reflected, and thus, there is a further term used, the reflectivity, which is equal to (1 – α).

Radiation between Two Bodies

The radiant energy transferred between two surfaces depends upon their temperatures, the geometric arrangement, and their emissivities. For two parallel surfaces, facing each other and neglecting edge effects, each must intercept the total energy emitted by the other, either absorbing or reflecting it. In this case, the net heat transferred from the hotter to the cooler surface is given by:

$$q = AC\sigma\,(\boldsymbol{T}_1^{\,4} - \boldsymbol{T}_2^{\,4}) \qquad (5.10)$$

where $1/C = 1/\varepsilon_1 + 1/\varepsilon_2 - 1$, ε_1 is the emissivity of the surface at temperature $\boldsymbol{T}_1$ and ε_2 is the emissivity of the surface at temperature $\boldsymbol{T}_2$.

Radiation to a Small Body from its Surroundings

In the case of a relatively small body in surroundings that are at a uniform temperature, the net heat exchange is given by the equation

$$q = A\varepsilon\sigma(\boldsymbol{T}_1^{\,4} - \boldsymbol{T}_2^{\,4}) \qquad (5.11)$$

where ε is the emissivity of the body, $\boldsymbol{T}_1$ is the absolute temperature of the body and $\boldsymbol{T}_2$ is the absolute temperature of the surroundings.

For many practical purposes in food process engineering, eqn. (5.11) covers the situation; for example for a loaf in an oven receiving radiation from the walls around it, or a meat carcass radiating heat to the walls of a freezing chamber.

In order to be able to compare the various forms of heat transfer, it is necessary to see whether an equation can be written for radiant-heat transfer similar to the general heat transfer eqn. (5.3). This means that for radiant-heat transfer:

$$q = h_rA(\boldsymbol{T}_1 - \boldsymbol{T}_2) = h_rA(T_1 - T_2) = h_rA\,\Delta T \qquad (5.12)$$

where h_r is the radiation heat transfer coefficient, $\boldsymbol{T}_1$ is the temperature of the body and $\boldsymbol{T}_2$ is the temperature of the surroundings. The $\boldsymbol{T}$ would normally be the absolute temperature for the radiation, but the absolute temperature difference is equal to the Celsius temperature difference, because 273 is added and subtracted and so $(\boldsymbol{T}_1 - \boldsymbol{T}_2) = (T_1 - T_2) = \Delta T$.

Equating eqn. (5.11) and eqn. (5.12)

$$q = h_rA(\boldsymbol{T}_1 - \boldsymbol{T}_2) = A\varepsilon\sigma\,(\boldsymbol{T}_1^{4} - \boldsymbol{T}_2^{4})$$

Therefore

$$h_r = \varepsilon\sigma\,(\boldsymbol{T}_1^{\,4} - \boldsymbol{T}_2^{4})/(\boldsymbol{T}_1 - \boldsymbol{T}_2)$$

$$= \varepsilon\sigma(\boldsymbol{T}_1 + \boldsymbol{T}_2)(\boldsymbol{T}_1^{2} + \boldsymbol{T}_2^{\,2})$$

If $\boldsymbol{T}_m = (\boldsymbol{T}_1 + \boldsymbol{T}_2)/2$, we can write $\boldsymbol{T}_1 + e = \boldsymbol{T}_m$ and $\boldsymbol{T}_2 - e = \boldsymbol{T}_m$

where

$$2e = \boldsymbol{T}_1 - \boldsymbol{T}_2$$

also

$$(\boldsymbol{T}_1 + \boldsymbol{T}_2) = 2\,\boldsymbol{T}_m$$

and then

$$(\boldsymbol{T}_1^2 + \boldsymbol{T}_2^2) = \boldsymbol{T}_m^2 - 2e\boldsymbol{T}_m + e^2 + \boldsymbol{T}_m^2 + 2e\boldsymbol{T}_m + e^2$$

$$= 2\boldsymbol{T}_m^2 + 2e^2$$

$$= 2\boldsymbol{T}_m^2 + (\boldsymbol{T}_1 - \boldsymbol{T}_2)^2/2$$

Therefore $\quad h_r = \varepsilon\sigma\,(2\boldsymbol{T}_m)[2\boldsymbol{T}_m^2 + (\boldsymbol{T}_1 - \boldsymbol{T}_2)^2/2]$

Now, if $(\boldsymbol{T}_1 - \boldsymbol{T}_2) \ll \boldsymbol{T}_1$ or $\boldsymbol{T}_2$, that is if the difference between the temperatures is small compared with the numerical values of the absolute temperatures, we can use the mean temperature, $\boldsymbol{T}_m$:

$$h_r \approx \varepsilon\sigma\, 4\boldsymbol{T}_m^3$$

and so

$$q = h_r A\, \Delta\boldsymbol{T}$$
$$= \varepsilon\sigma\, 4\boldsymbol{T}_m^3 A\, \Delta\boldsymbol{T}$$
$$= (\varepsilon \times 5.73 \times 10^{-8} \times 4 \times \boldsymbol{T}_m^3 A\, \Delta\boldsymbol{T})$$
$$q = 0.23\varepsilon\,(\boldsymbol{T}_m/100)^3 A\, \Delta\boldsymbol{T} \qquad (5.13)$$

EXAMPLE 5.6.Radiation heat transfer to loaf of bread in an oven

Calculate the net heat transfer by radiation to a loaf of bread in an oven at a uniform temperature of 177°C, if the emissivity of the surface of the loaf is 0.85, using eqn. (5.11). Compare this result with that obtained by using eqn. (5.13). The total surface area and temperature of the loaf are respectively 0.0645 m^2 and 100°C.

$$q = A\varepsilon\sigma\,(\boldsymbol{T}_1^{\,4} - \boldsymbol{T}_2^{\,4})$$

$$= 0.0645 \times 0.85 \times 5.73 \times 10^{-8}\,(450^4 - 373^4)$$
$$= 68.0 Js^{-1}.$$

By eqn. (5.13)

$$q = 0.23\varepsilon\,(\boldsymbol{T}_m/100)^3 A\Delta\boldsymbol{T}$$

$$= 0.23 \times 0.85(411/100)^3 \times 0.0645 \times 77$$
$$= \underline{67.4 Js^{-1}}.$$

Notice that even with quite a large temperature difference, eqn. (5.13) gives a close approximation to the result obtained using eqn. (5.11).

CONVECTION HEAT TRANSFER

Convection heat transfer is the transfer of energy by the mass movement of groups of molecules. It is restricted to liquids and gases, as mass molecular movement does not occur at an appreciable speed in solids. It cannot be mathematically predicted as easily as can transfer by conduction or radiation and so its study is largely based on experimental results rather than on theory. The most satisfactory convection heat transfer formulae are relationships between dimensionless groups of physical quantities. Furthermore, since the laws of molecular transport govern both heat flow and viscosity, convection heat transfer and fluid friction are closely related to each other.

Convection coefficients will be studied under two sections, firstly, natural convection in which movements occur due to density differences on heating or cooling; and secondly, forced convection, in which an external source of energy is applied to create movement. In many practical cases, both mechanisms occur together.

Natural Convection

Heat transfer by natural convection occurs when a fluid is in contact with a surface hotter or colder than itself. As the fluid is heated or cooled it changes its density. This difference in density causes movement in the fluid that has been heated or cooled and causes the heat transfer to continue.

There are many examples of natural convection in the food industry. Convection is significant when hot surfaces, such as retorts which may be vertical or horizontal cylinders, are exposed with or without insulation to colder ambient air. It occurs when food is placed inside a chiller or freezer store in which circulation is not assisted by fans. Convection is important when material is placed in ovens without fans and afterwards when the cooked material is removed to cool in air.

It has been found that natural convection rates depend upon the physical constants of the fluid, density ρ, viscosity μ, thermal conductivity k, specific heat at constant pressure c_p and coefficient of thermal expansion β (beta) which for gases = l/T by Charles' Law. Other factors that also affect convection-heat transfer are, some linear dimension of the system, diameter D or length L, a temperature difference term, ΔT, and the gravitational acceleration g since it is density differences acted upon by gravity that create circulation. Heat transfer rates are expressed in terms of a convection heat transfer coefficient (h_c), which is part of the general surface coefficient h_s, in eqn. (5.5).

Experimentally, if has been shown that convection heat transfer can be described in terms of these factors grouped in dimensionless numbers which are known by the names of eminent workers in this field:

Nusselt number (Nu) = $(h_c D/k)$
Prandtl number (Pr) = $(c_p \mu /k)$
Grashof number (Gr) = $(D^3 \rho^2 g\ \beta\ \Delta T /\mu^2)$

and in some cases a length ratio (L/D).

If we assume that these ratios can be related by a simple power function we can then write the most general equation for natural convection:

$$(\mathrm{Nu}) = K(\mathrm{Pr})^k(\mathrm{Gr})^m(L/D)^n \qquad (5.14)$$

Experimental work has evaluated K, k, m, n, under various conditions. For a discussion, see McAdams (1954). Once K, k, m, n are known for a particular case, together with the appropriate physical characteristics of the fluid, the Nusselt number can be calculated. From the Nusselt number we can find h_c and so determine the rate of convection heat transfer by applying eqn. (5.5). In natural convection equations, the values of the physical constants of the fluid are taken at the mean temperature between the surface and the bulk fluid. The Nusselt and Biot numbers look similar: they differ in that for Nusselt, k and h both refer to the fluid, for Biot k is in the solid and h is in the fluid.

Natural Convection Equations

These are related to a characteristic dimension of the body (food material for example) being considered, and typically this is a length for rectangular bodies and a diameter for spherical/cylindrical ones

(1) *Natural convection about vertical cylinders and planes, such as vertical retorts and oven walls*

$$(\mathrm{Nu}) = 0.53(\mathrm{Pr.Gr})^{0.25} \quad \text{for } 10^4 < (\mathrm{Pr.Gr}) < 10^9 \qquad (5.15)$$

$$(\mathrm{Nu}) = 0.12(\mathrm{Pr.Gr})^{0.33} \quad \text{for } 10^9 < (\mathrm{Pr.Gr}) < 10^{12} \qquad (5.16)$$

For air, these equations can be approximated respectively by:

$$h_c = 1.3(\Delta T/L)^{0.25} \qquad (5.17)$$

$$h_c = 1.8(\Delta T)^{0.25} \qquad (5.18)$$

Equations (5.17) and (5.18) are dimensional equations and are in standard units [ΔT in ^{o}C, L (or D) in metres and h_c in $Jm^{-2}s^{-1}{}^{o}C^{-1}$]. The characteristic dimension to be used in the calculation of (Nu) and (Gr) in these equations is the height of the plane or cylinder.

(2) *Natural convection about horizontal cylinders such as a steam pipe or sausages lying on a rack*

$$(Nu) = 0.54(Pr.Gr)^{0.25} \text{ for laminar flow in range } 10^3 < (Pr.Gr) < 10^9. \quad (5.19)$$

Simplified equations can be employed in the case of air, which is so often encountered in contact with hotter or colder foods giving again:

for $10^4 < (Pr.Gr) < 10^9$ $\quad h_c = 1.3(\Delta T/D)^{0.25}$ (5.20)

and for $10^9 < (Pr.Gr) < 10^{12}$ $\quad h_c = 1.8(\Delta T)^{0.33}$ (5.21)

(3) *Natural convection from horizontal planes, such as slabs of cake cooling*

The corresponding cylinder equations may be used, employing the length of the plane instead of the diameter of the cylinder whenever *D* occurs in (Nu) and (Gr). In the case of horizontal planes, cooled when facing upwards, or heated when facing downwards, which appear to be working against natural convection circulation, it has been found that half of the value of h_c in eqns. (5.19) - (5.21) corresponds reasonably well with the experimental results.

Note carefully that the simplified equations are dimensional. Temperatures must be in °C and lengths in m and then h_c will be in $Jm^{-2}s^{-1}\,{}^oC^{-1}$. Values for σ, k and μ are measured at the film temperature, which is midway between the surface temperature and the temperature of the bulk liquid.

EXAMPLE 5.7. Heat loss from a cooking vessel
Calculate the rate of convection heat loss to ambient air from the side walls of a cooking vessel in the form of a vertical cylinder 0.9m in diameter and 1.2m high. The outside of the vessel insulation, facing ambient air, is found to be at 49°C and the air temperature is 17°C.

First it is necessary to establish the value of (Pr.Gr).
From the properties of air, at the mean film temperature, (49 + 17)/2, that is 33°C,
$\mu = 1.9 \times 10^{-5}$ Nsm $^{-2}$, $c_p = 1.0 kJkg^{-1}\,{}^oC^{-1}$, $k = 0.025 Jm^{-1}s^{-1}\,{}^oC^{-1}$, $\beta = 1/308$, $\rho = 1.12 kgm^{-3}$.
From the conditions of the problem, characteristic dimension = height = 1.2 m, $\Delta T = 32^oC$.

$$(Pr.Gr) = (c_p\mu/k)(D^3\rho^2 g\,\beta\,\Delta T/\mu^2)$$

$$= (L^3\rho^2 g\,\beta\,\Delta T\,c_p)/(\mu\,k)$$

$$= [(1.2)^3 \times (1.12)^2 \times 9.81 \times 1 \times 32 \times 1.0 \times 10^3)/(308 \times 1.9 \times 10^{-5} \times 0.025)$$

$$= 5 \times 10^9$$

Therefore eqn. (5.18) is applicable.

and so $\quad h_c = 1.8\Delta T^{0.25} = 1.8(32)^{0.25}$
$\quad = 4.3\ Jm^{-2}s^{-1}\,{}^oC^{-1}$

Total area of vessel wall A $= \pi DL = \pi \times 0.9 \times 1.2 = 3.4 m^2$

ΔT $= 32^oC$.

Therefore heat loss rate $= h_c A(T_1 - T_2)$

$= 4.3 \times 3.4 \times 32$

$= \underline{468 \ Js^{-1}}$

Forced Convection

When a fluid is forced past a solid body and heat is transferred between the fluid and the body, this is called forced convection heat transfer. Examples in the food industry are in the forced-convection ovens for baking bread, in blast and fluidized freezing, in ice-cream hardening rooms, in agitated retorts, in meat chillers. In all of these, foodstuffs of various geometrical shapes are heated or cooled by a surrounding fluid, which is moved relative to them by external means.

The fluid is constantly being replaced, and the rates of heat transfer are, therefore, higher than for natural convection. Also, as might be expected, the higher the velocity of the fluid the higher the rate of heat transfer. In the case of low velocities, where rates of natural convection heat transfer are comparable to those of forced convection heat transfer, the Grashof number is still significant. But in general the influence of natural circulation, depending as it does on coefficients of thermal expansion and on the gravitational acceleration, is replaced by dependence on circulation velocities and the Reynolds number.

As with natural convection, the results are substantially based on experiment and are grouped to deal with various commonly met situations such as fluids flowing in pipes, outside pipes, etc.

Forced Convection Equations

(1) *Heating and cooling inside tubes, generally fluid foods being pumped through pipes*

In cases of moderate temperature differences and where tubes are reasonably long, for laminar flow it is found that:

$$(Nu) = 4 \tag{5.22}$$

and where turbulence is developed for (Re) > 2100 and (Pr) > 0.5

$$(Nu) = 0.023(Re)^{0.8}(Pr)^{0.4} \tag{5.23}$$

For more viscous liquids, such as oils and syrups, the surface heat transfer will be affected, depending upon whether the fluid is heating or being cooled. Under these cases, the viscosity effect can be allowed, for (Re)> 10,000, by using the equation:

$$(Nu) = 0.027(\mu/\mu_s)^{0.14}(Re)^{0.8}(Pr)^{0.33} \quad (5.24)$$

In both cases, the fluid properties are those of the bulk fluid except for μ_s which is the viscosity of the fluid at the temperature of the tube surface.

Since (Pr) varies little for gases, either between gases or with temperature, it can be taken as 0.75 and eqn. (5.23) simplifies for gases to:

$$(Nu) = 0.02(Re)^{0.8}. \quad (5.25)$$

In this equation the viscosity ratio is assumed to have no effect and all quantities are evaluated at the bulk gas temperature. For other factors constant, this becomes $h_c = k'v^{0.8}$ as in equation 5.28.

(2) *Heating or cooling over plane surfaces*

Many instances of foods approximate to plane surfaces, such as cartons of meat or ice cream or slabs of cheese. For a plane surface, the problem of characterizing the flow arises, as it is no longer obvious what length to choose for the Reynolds number. It has been found, however, that experimental data correlate quite well if the length of the plate measured in the direction of the flow is taken for *D* in the Reynolds number and the recommended equation is:

$$(Nu) = 0.036\,(Re)^{0.8}(Pr)^{0.33} \qquad \text{for } (Re) > 2 \times 10^4 \quad (5.26)$$

For the flow of air over flat surfaces simplified equations are:

$$h_c = 5.7 + 3.9v \quad \text{for } v < 5\text{ms}^{-1} \quad (5.27)$$

$$h_c = 7.4v^{0.8} \qquad \text{for } 5 < v < 30\text{ms}^{-1} \quad (5.28)$$

These again are dimensional equations and they apply only to smooth plates. Values for h_c for rough plates are slightly higher.

(3) *Heating and cooling outside tubes*

Typical examples in food processing are water chillers, chilling sausages, processing spaghetti. Experimental data in this case have been correlated by the usual form of equation:

$$(Nu) = K\,(Re)^n(Pr)^m \quad (5.29)$$

The powers n and m vary with the Reynolds number. Values for *D* in (Re) are again a difficulty and the diameter of the tube, over which the flow occurs, is used. It should be noted that in this case the same values of (Re) can not be used to denote streamline or turbulent conditions as for fluids flowing inside pipes.

For gases and for liquids at high or moderate Reynolds numbers:

$$(\mathrm{Nu}) = 0.26(\mathrm{Re})^{0.6}(\mathrm{Pr})^{0.3} \qquad (5.30)$$

whereas for liquids at low Reynolds numbers, 1 < (Re) < 200:

$$(\mathrm{Nu}) = 0.86(\mathrm{Re})^{0.43}(\mathrm{Pr})^{0.3} \qquad (5.31)$$

As in eqn. (5.23), (Pr) for gases is nearly constant so that simplified equations can be written. Fluid properties in these forced convection equations are evaluated at the mean film temperature, which is the arithmetic mean temperature between the temperature of the tube walls and the temperature of the bulk fluid.

EXAMPLE 5.8. Heat transfer in water flowing over a sausage
Water is flowing at 0.3 ms^{1} across a 7.5cm diameter sausage at 74°C. If the bulk water temperature is 24°C, estimate the heat-transfer coefficient.

Mean film temperature = (74 + 24)/2 = 49°C.

Properties of water at 49°C are taken as:
$c_p = 4.186 kJkg^{-1}{}^{o}C^{-1}$, $k = 0.64 Jm^{-1}s^{-1}{}^{o}C^{-1}$, $\mu = 5.6 x\ 10^{-4} Nsm^{-2}$, $\rho = 1000 kgm^{-3}$.

Therefore (Re) $= (Dv\rho/\mu)$
$= (0.075 \times 0.3 \times 1000)/(5.6 \times 10^{-4})$
$= 4.02 \times 10^4$
$(\mathrm{Re})^{0.6} = 580$

(Pr) $= (c_p \mu / k)$
$= (4186 \times 5.6 \times 10^{-4})/0.64$
$= 3.66.$
$(\mathrm{Pr})^{0.3} = 1.48$

(Nu) $= (h_c D/k)$
$= 0.26(\mathrm{Re})^{0.6}(\mathrm{Pr})^{0.3}$

Therefore $h_c = k/D \times [0.26 \times (\mathrm{Re})^{0.6}(\mathrm{Pr})^{0.3}]$
$= (0.64 \times 0.26 \times 580 \times 1.48)/0.075$
$= \underline{1904\ Jm^{-2}s^{-1}{}^{o}C^{-1}}$

EXAMPLE 5.9. Surface heat transfer to vegetable puree
Calculate the surface heat transfer coefficient to a vegetable puree, which is flowing at an estimated 3m min^{-1}over a flat plate 0.9m long by 0.6m wide. Steam is condensing on the other side of the plate and maintaining the surface, which is in contact with the puree, at 104°C. Assume that the properties of the vegetable puree are, density $1040 kgm^{-3}$, specific heat $3980 Jkg^{-1}{}^{o}C^{-1}$, viscosity 0.002 Nsm^{-2}**,** thermal conductivity 0.52 $Jm^{-1}s^{-1}{}^{o}C^{-1}$.

v = 3m min^{-1} = 3/60 ms^{-1}
(Re) $= (Lv\rho/\mu)$

$= (0.9 \times 3 \times 1040)/(2 \times 10^{-3} \times 60)$
$= 2.34 \times 10^{4}$

Therefore eqn. (5.26) is applicable and so:

$Nu = (h_c L/k) = 0.036(Re)^{0.8}(Pr)^{0.33}$

$Pr = (c_p \mu /k)$
$= (3980 \times 2 \times 10^{-3})/0.52$
$= 15.3$

and so

$(h_c L/k) = 0.036(2.34 \times 10^4)^{0.8} 15.3^{0.33}$

$h_c = (0.52 \times 0.036)(3.13 \times 10^3)(2.46)/0.9$
$= \underline{160 \text{ J} m^{-2} s^{-1} {}^oC^{-1}}$

EXAMPLE 5.10. Heat loss from a cooking vessel

What would be the rate of heat loss from the cooking vessel of Example 5.7, if a draught caused the air to move past the cooking vessel at a speed of 61m min^{-1}

Assuming the vessel is equivalent to a flat plate then from eqn. (5.27)

$v = 61/60 = 1.02 \text{ m } s^{-1}$ that is $v < 5 \text{ m } s^{-1}$
$h_c = 5.7 + 3.9v$
$= 5.7 + (3.9 \times 61)/60$
$= 9.7 \text{ J} m^{-2} s^{-1} {}^oC^{-1}$

So with $A = 3.4m^2$, $\Delta T = 32^oC$,

$q = 9.7 \times 3.4 \times 32$
$= \underline{1055 \text{ J} s^{-1}}$

OVERALL HEAT TRANSFER COEFFICIENTS

It is most convenient to use overall heat transfer coefficients in heat transfer calculations as these combine all of the constituent factors into one, and are based on the overall temperature drop. An overall coefficient, *U*, combining conduction and surface coefficients, has already been introduced in eqn. (5.5). Radiation coefficients, subject to the limitations discussed in the section on radiation, can be incorporated also in the overall coefficient. The radiation coefficients should be combined with the convection coefficient to give a total surface coefficient, as they are in series, and so:

$$h_s = (h_r + h_c) \qquad (5.32)$$

The overall coefficient *U* for a composite system, consisting of surface film, composite wall, surface film, in series, can then be calculated as in eqn. (5.5) from:

$$1/U = 1/(h_r + h_c)_1 + x_1/k_1 + x_2/k_2 + \ldots\ldots + 1/(h_r + h_c)_2. \qquad (5.33)$$

EXAMPLE 5.11. Effect of air movement on heat transfer in a cold store

In Example 5.2, the overall conductance of the materials in a cold-store wall was calculated. Now on the outside of such a wall a wind of 6.7 ms^{-1} is blowing, and on the inside a cooling unit moves air over the wall surface at about 0.61 ms^{-1}. The radiation coefficients can be taken as 6.25 and 1.7 $Jm^{-2}s^{-1}{}^{o}C^{-1}$ on the outside and inside of the wall respectively. Calculate the overall heat transfer coefficient for the wall.

On outside surface: $v = 6.7 ms^{-1}$.

From eqn. (5.28)

$$h_c = 7.4v^{0.8} = 7.4(6.7)^{0.8}$$
$$= 34\ Jm^{-2}s^{-1}{}^{o}C^{-1}$$

and $h_r = 6.25\ Jm^{-2}s^{-1}{}^{o}C^{-1}$

Therefore $h_{s1} = (34+6) = 40\ Jm^{-2}s^{-1}{}^{o}C^{-1}$

On inside surface: $v = 0.61\ ms^{-1}$.

From eqn. (5.27)

$$h_c = 5.7 + 3.9v = 5.7 + (3.9 \times 0.61)$$
$$= 8.1\ Jm^{-2}s^{-1}{}^{o}C^{-1}$$

and $h_r = 1.7\ Jm^{-2}s^{-1}{}^{o}C^{-1}$

Therefore $h_{s2} = (8.1 + 1.7) = 9.8\ Jm^{-2}s^{-1}{}^{o}C^{-1}$

Now from Example 5.2 the overall conductance of the wall,

$$U_{old} = 0.38\ Jm^{-2}s^{-1}\ {}^{o}C^{-1}$$

and

$$1/U_{new} = 1/h_{s1} + 1/U_{old} + 1/h_{s2}$$
$$= 1/40 + 1/0.38 + 1/9.8$$
$$= 2.76.$$

Therefore $U_{new} = \underline{0.36\ Jm^{-2}s^{-1}\ {}^{o}C^{-1}}$

In eqn. (5.33) often one or two terms are much more important than other terms because of their numerical values. In such a case, the important terms, those signifying the low thermal conductances, are said to be the controlling terms. Thus, in Example 5.11 the introduction of values for the surface coefficients made only a small difference to the overall *U* value for the insulated wall. The reverse situation might be the case for other walls that were better heat conductors.

EXAMPLE 5.12. Comparison of heat transfer in brick and aluminium walls

Calculate the respective *U* values for a wall made from either (a) 10 cm of brick of thermal conductivity 0.7 $Jm^{-1}s^{-1}\ {}^{o}C^{-1}$, or (b) 1.3mm of aluminium sheet, conductivity 208 $Jm^{-1}s^{-1}{}^{o}C^{-1}$. Surface heat-transfer coefficients are on the one side 9.8 and on the other 40 $Jm^{-2}s^{-1}{}^{o}C^{-1}$.

(a) For brick

$$k = 0.7\ Jm^{-1}s^{-1}\ {}^{o}C^{-1}$$
$$x/k = 0.1/0.7 = 0.14$$

Therefore $1/U = 1/40+0.14+1/9.8$

$= 0.27$

$U = \underline{3.7\ Jm^{-2}s^{-1}\ {}^{o}C^{-1}}$

(b) For aluminium

$k = 208\ Jm^{-1}s^{-1}\ {}^{o}C^{-1}$

$x/k = 0.0013/208$

$= 6.2 \times 10^{-6}$

$1/U = 1/40 + 6.2 \times 10^{-6} + 1/9.8$

$= 0.13$

$U = \underline{7.7\ Jm^{-2}s^{-1}\ {}^{o}C^{-1}}$

Comparing the calculations in Example 5.11 with those in Example 5.12, it can be seen that the relative importance of the terms varies. In the first case, with the insulated wall, the thermal conductivity of the insulation is so low that the neglect of the surface terms makes little difference to the calculated *U* value. In the second case, with a wall whose conductance is of the same order as the surface coefficients, all terms have to be considered to arrive at a reasonably accurate *U* value. In the third case, with a wall of high conductivity, the wall conductance is insignificant compared with the surface terms and it could be neglected without any appreciable effect on *U*. The practical significance of this observation is that if the controlling terms are known, then in any overall heat transfer situation other factors may often be neglected without introducing significant error. On the other hand, if all terms are of the same magnitude, there are no controlling terms and all factors have to be taken into account.

HEAT TRANSFER FROM CONDENSING VAPOURS

The rate of heat transfer obtained when a vapour is condensing to a liquid is very often important. In particular, it occurs in the food industry in steam-heated vessels where the steam condenses and gives up its heat; and in distillation and evaporation where the vapours produced must be condensed. In condensation, the latent heat of vaporization is given up at constant temperature, the boiling temperature of the liquid.

Two generalized equations have been obtained:

(1) *For condensation on vertical tubes or plane surfaces*

$$h_v = 0.94[(k^3\rho^2 g/\mu) \times (\lambda/L\Delta T)]^{0.25} \qquad (5.34)$$

where λ(lambda) is the latent heat of the condensing liquid in J kg^{-1}, *L* is the height of the plate or tube and the other symbols have their usual meanings.

(2) *For condensation on a horizontal tube*

$$h_h = 0.72[(k^3\rho^2 g/\mu) \times (\lambda/D\Delta T)]^{0.25} \qquad (5.35)$$

where D is the diameter of the tube.

These equations apply to condensation in which the condensed liquid forms a film on the condenser surface. This is called film condensation: it is the most usual form and is assumed to occur in the absence of evidence to the contrary.

However, in some cases the condensation occurs in drops that remain on the surface and then fall off without spreading a condensate film over the whole surface. Since the condensate film itself offers heat transfer resistance, film condensation heat transfer rates would be expected to be lower than drop condensation heat-transfer rates and this has been found to be true. Surface heat transfer rates for drop condensation may be as much as ten times as high as the rates for film condensation.

The contamination of the condensing vapour by other vapours, which do not condense under the condenser conditions, can have a profound effect on overall coefficients. Examples of a non-condensing vapour are air in the vapours from an evaporator and in the jacket of a steam pan. The adverse effect of non-condensable vapours on overall heat transfer coefficients is due to the difference between the normal range of condensing heat transfer coefficients, 1200 - 12,000 $Jm^{-2}s^{-1o}C^{-1}$, and the normal range of gas heat transfer coefficients with natural convection or low velocities, of about 6 $Jm^{-2}s^{-1o}C^{-1}$.

Uncertainties make calculation of condensation coefficients difficult, and for many purposes it is near enough to assume the following coefficients:

for condensing steam	12,000 $Jm^{-2}s^{-1o}C^{-1}$
for condensing ammonia	6,000 $Jm^{-2}s^{-1o}C^{-1}$
for condensing organic liquids	1,200 $Jm^{-2}s^{-1o}C^{-1}$

The heat transfer coefficient for steam with 3% air falls to about 3500 $Jm^{-2}s^{-1o}C^{-1}$, and with 6% air to about 1200 $Jm^{-2}s^{-1o}C^{-1}$.

EXAMPLE 5.13. Condensing ammonia in a refrigeration plant

A steel tube of 1mm wall thickness is being used to condense ammonia, using cooling water outside the pipe in a refrigeration plant. If the water-side heat transfer coefficient is estimated at 1750 $Jm^{-2}s^{-1o}C^{-1}$ and the thermal conductivity of steel is 45 $Jm^{-1}s^{-1o}C^{-1}$, calculate the overall heat-transfer coefficient.

Assuming the ammonia condensing coefficient, 6000 $Jm^{-2}s^{-1o}C^{-1}$

$$1/U = 1/h_1 + x/k + 1/h_2$$
$$= 1/1750 + 0.001/45 + 1/6000$$
$$= 7.6 \times 10^{-4}$$

$$U = \underline{1300\ Jm^{-2}s^{-1o}C^{-1}}.$$

HEAT TRANSFER TO BOILING LIQUIDS

When the presence of a heated surface causes a liquid near it to boil, the intense agitation gives rise to high local coefficients of heat transfer. A considerable amount of experimental work has been carried out on this, but generalized correlations are still not very adequate. It has been found that the apparent coefficient varies considerably with the temperature difference between the heating surface and the liquid. For temperature differences greater than about 20°C, values of h decrease, apparently because of blanketing of the heating surface by vapours. Over the range of temperature differences from 1°C to 20°C, values of h for boiling water increase from 1200 to about 60,000 $Jm^{-2}s^{-1}°C^{-1}$. For boiling water under atmospheric pressure, the following equation is approximately true:

$$h = 50(\Delta T)^{2.5} \tag{5.36}$$

where ΔT is the difference between the surface temperature and the temperature of the boiling liquid and it lies between 2°C and 20°C.

In many applications the high boiling film coefficients are not of much consequence, as resistance in the heat source controls the overall coefficients.

SUMMARY

1. Heat is transferred by conduction, radiation and convection.

2. Heat-transfer rates are given by the general equation:

$$q = UA\Delta T$$

3. For heat conduction:

$$q = (k/x)A\ \Delta T$$

4. For radiation:

$$q = A\varepsilon\sigma\ (\boldsymbol{T}_1^{\ 4} - \boldsymbol{T}_2^{4})$$

5. Overall heat-transfer coefficients are given by:

   (a) for heat conductances in series,

$$1/U \quad = x_1/k_1 + x_2/k_2 + \ \ldots\ldots\ldots..$$

   (b) for radiation convection and conduction,

$$1/U \quad = 1/(h_{r1} + h_{c1}) + x_1/k_1 + x_2/k_2 + \ldots\ldots + 1/(h_{r2} + h_{c2})$$

6. For convection heat-transfer coefficients are given by equations of the general form:

$$(\mathrm{Nu}) = K(\mathrm{Pr})^{k}(\mathrm{Gr})^{m}(L/D)^{n}$$

for natural convection

$$\mathrm{Nu} = K(\mathrm{Pr.Gr})^{o}$$

for forced convection.

$$\mathrm{Nu} = K\ (\mathrm{Re})^{p}(\mathrm{Pr})^{q}$$

PROBLEMS

1. It is desired to limit the heat loss from a wall of polystyrene foam to 8 Js^{-1} when the temperature on one side is 20^oC and on the other -18^oC. The area is $1m^2$. How thick should the polystyrene be?

   (17cm)

2. Calculate the overall heat-transfer coefficient from air to a product packaged in 3.2mm of solid cardboard, and 0.1mm of celluloid, if the surface air heat transfer coefficient is 11 $Jm^{-2}s^{-1}{}^oC^{-1}$.

   (7.29 $Jm^{-2}s^{-1}{}^oC^{-1}$)

3. The walls of an oven are made from steel sheets with insulating board between them of thermal conductivity 0.l8 $Jm^{-1}s^{-1}{}^oC^{-1}$. If the maximum internal temperature in the oven is 300^oC and the outside surface of the oven wall must not rise above 50^oC, estimate the minimum necessary thickness of insulation assuming surface heat transfer coefficients to the air on both sides of the wall are 15 $Jm^{-2}s^{-1}{}^oC^{-1}$. Assume the room air temperature outside the oven to be 25^oC and that the insulating effect of the steel sheets can be neglected.

   (10.8cm)

4. Calculate the thermal conductivity of uncooked pastry. Measurements show that with a temperature difference of 17^oC across a large slab 1.3cm thick, the heat flow is $0.5Js^{-1}$ through an area of $10cm^2$ of slab surface.

   (0.38 $Jm^{-1}s^{-1}{}^oC^{-1}$)

5. Thick soup is being boiled in a pan and because of inadequacy of stirring a layer of soup builds up on the bottom of the pan to a thickness of 2mm. The hot plate on which the pan is standing is at an average temperature of 500^oC, the heat transfer coefficient from the plate to the pan is 600 $Jm^{-2}s^{-1}{}^oC^{-1}$, and that from the soup layer to the surface of the bulk soup is 1400 $Jm^{-2}s^{-1}{}^oC^{-1}$. The pan is of aluminium 2mm thick. Find the temperature between the layer of soup and the pan surface. Assume the thermal conductivity of the soup layer approximates that of water.

   (392^oC)

6. Peas are being blanched by immersing them in hot water at 85^oC until the centre of the pea reaches 70^oC. The average pea diameter is 0.0048m and the thermal properties of the peas are: thermal conductivity $0.48Jm^{-1}s^{-1}{}^oC^{-1}$, specific heat 3.51 x $10^3Jkg^{-1}{}^oC^{-1}$ and density $990kgm^{-3}$. The surface heat transfer coefficient to the peas was estimated as $400Jm^{-2}s^{-1}{}^oC^{-1}$. (a) How long should it take the average pea to reach 70^oC if its initial temperature was 18^oC just prior to immersion? (b) If the diameter of the largest pea is 0.0063m, what temperature will its centre have reached when that of the average pea is 70^oC?

   ((a) 19.2s (b) 55^oC)

7. Some people believe that because of' its lower thermal conductivity stainless steel is appreciably thermally inferior to copper or mild steel as constructional material for a steam-

jacketed pan to heat food materials. The condensing heat transfer coefficient for the steam and the surface boiling coefficient on the two sides of the heating surface are respectively 10,000 $Jm^{-2}s^{-1o}C^{-1}$ and 700 $Jm^{-2}s^{-1o}C^{-1}$. The thickness of all three metal walls is 1.6mm. Compare the heating rates from all three constructions, assuming steady state conditions.

(Mild steel 2% worse than copper, stainless steel 4.5% worse than copper)

8. A long cylinder of solid aluminium 7.5cm in diameter initially at a uniform temperature of 5°C, is hung in an air blast at 100°C. If it is found that the temperature at the centre of the cylinder rises to 47.5°C after a time of 85 seconds, estimate the surface heat transfer coefficient from the cylinder to the air. Assume that the likely heat transfer coefficient is around 20 $Jm^{-2}s^{-1o}C^{-1}$ and that the cylinder is long enough to neglect the ends.

   (25 $Jm^{-2}s^{-1o}C^{-1}$)

9. A can of pumpkin puree 8.73cm diameter by 11.43cm in height is being heated in a steam retort in which the steam pressure is 100 kPa above atmospheric pressure. The pumpkin has a thermal conductivity of 0.83$Jm^{-1}s^{-1o}C^{-1}$, a specific heat of 3770$Jkg^{-1o}C^{-1}$ and a density of 1090kgm^{-3}. The temperature in the can prior to retorting was 20°C. Plot out the temperature at the centre of the can as a function of time until this temperature reaches 115°C. Assume *h* is very high.

   (at 70mins 111°C, at 79mins 115°C, at 80mins 116°C)

10. A steam boiler can be represented by a vertical cylindrical vessel 1.1m diameter and 1.3m high. It is maintained internally at a steam pressure of l50kPa. Estimate the energy savings that would result from insulating the vessel with a 5cm thick layer of mineral wool assuming heat transfer from the surface is by natural convection. The air temperature of the surroundings is 18°C and the thermal conductivity of the insulation is 0.04 $Jm^{-1}s^{-1o}C^{-1}$.

    (83%)

11. It is desired to chill 3m^3 of water per hour by means of horizontal coils in which ammonia is evaporated. The steel coils are 2.13cm outside diameter and 1.71cm inside diameter and the water is pumped across the outside of these at a velocity of 0.8 ms^{-1}. Estimate the length of pipe coil needed, if the mean temperature difference between the refrigerant and the water is 8°C, the mean temperature of the water is 4°C and the temperature of the water is decreased by 15°C in the chiller.

    (53.1m)